MATHEMATICS

NATIONAL 5 NOTES

$\times$ $\pm$

$\div$ $A = \pi r^2$ $\leq$

Σ x

π

$V = lbh$ $\surd$

$+$ $-$

INDEX:

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FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Sine rule: $\dfrac{\mathrm{a}}{\sin\mathrm{A}} = \dfrac{\mathrm{b}}{\sin\mathrm{B}} = \dfrac{\mathrm{c}}{\sin\mathrm{C}}$

Cosine rule: $\mathrm{a}^2 = \mathrm{b}^2 + \mathrm{c}^2 - 2bc\cos A$ or $\cos\mathrm{A} = \dfrac{\mathrm{b}^2 + \mathrm{c}^2 - \mathrm{a}^2}{2bc}$

Area of a triangle: Area $= \frac{1}{2}ab\sin C$

Volume of a sphere: Volume $= \frac{4}{3}\pi r^3$

Volume of a cone: Volume $= \frac{1}{3}\pi r^2 h$

Volume of a pyramid: Volume $= \frac{1}{3}Ah$

Volume of a cylinder: Volume $= \pi r^2 h$

Standard deviation: $s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n-1}}$, where n is the sample size.

CHAPTER 1: FRACTIONS

Simplify (cancel down):

$$\frac{28}{36} = \frac{28 \div 4}{36 \div 4} = \frac{7}{9}$$

divide by ***highest common factor***, $HCF(28,36) = 4$

Mixed numbers and **improper fractions** (top heavy):

$$4 \times \mathbf{2} + 3 = 11 \quad \longrightarrow$$

$$\mathbf{2}\frac{3}{4} = \mathbf{2} + \frac{3}{4} = \frac{\mathbf{8}}{\mathbf{4}} + \frac{3}{4} = \frac{11}{4}$$

'bottom' stays as 4

$$\longleftarrow \quad 11 \div 4 = \mathbf{2}\ R3$$

Addition and Subtraction requires a common denominator:

$$\frac{5}{6} - \frac{4}{9}$$

least common multiple *LCM(6,9)=18*

$$\frac{5 \times 3}{6 \times 3} = \frac{15}{18} \qquad \frac{4 \times 2}{9 \times 2} = \frac{8}{18}$$

both fractions now 18ths

$$= \frac{15}{18} - \frac{8}{18}$$

$$= \frac{7}{18}$$

subtract 'top' numbers, still 18ths

Multiplication and Division requires **no** mixed numbers:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

$$\frac{a}{b} \div \frac{\mathbf{c}}{\mathbf{d}} = \frac{a}{b} \times \frac{\mathbf{d}}{\mathbf{c}}$$

$$\frac{3}{10} \times 2\frac{3}{4}$$

$$= \frac{3}{10} \times \frac{11}{4}$$

go 'top heavy'

$$= \frac{33}{40}$$

$$\frac{4}{5} \div \frac{7}{8}$$

$$= \frac{4}{5} \times \frac{8}{7}$$

multiply by reciprocal

$$= \frac{32}{35}$$

ORDER OF CALCULATION: $\times$ *and* $\div$ *before* $+$ *or* $-$

to change the order use brackets *BRACKETS FIRST*!

Carry out separate calculations:

(1) $7\frac{5}{6} - \frac{8}{9}$ *of* $2\frac{3}{4}$

multiply first

$\frac{8}{9} \times \frac{11}{4}$ ***of** means **multiply***

$= \frac{88}{36}$ *fully simplify*

$= \frac{22}{9}$

$= 2\frac{4}{9}$

subtraction last

$7\frac{5}{6} - 2\frac{4}{9}$

$= 7\frac{15}{18} - 2\frac{8}{18}$ *common denominator*

$= 5\frac{7}{18}$ *subtract whole numbers and fractions separately*

(2) $\left(\frac{5}{6} - \frac{1}{2}\right) \div 1\frac{3}{4}$

$\frac{5}{6} - \frac{1}{2}$ *brackets first*

$= \frac{5}{6} - \frac{3}{6}$

$= \frac{2}{6}$ *fully simplify*

$= \frac{1}{3}$

$\frac{1}{3} \div 1\frac{3}{4}$

$= \frac{1}{3} \div \frac{7}{4}$ *'top-heavy' first*

$= \frac{1}{3} \times \frac{4}{7}$ *multiply by reciprocal*

$= \frac{4}{21}$

CHAPTER 2: PERCENTAGES

PERCENTAGE CHANGE

	original value		changed value
INCREASE: growth, appreciation, compound interest	100%	$\xrightarrow{+a\%}$	$(100+a)\%$
DECREASE: decay, depreciation	100%	$\xrightarrow{-a\%}$	$(100-a)\%$

For example,

8% increase: $100\% \xrightarrow{+8\%} 108\% = 1\cdot 08$ multiply quantity by $1\cdot 08$ for 8% increase

8% decrease: $100\% \xrightarrow{-8\%} 92\% = 0\cdot 92$ multiply quantity by $0\cdot 92$ for 8% decrease

REVERSING PERCENTAGE CHANGE

Divide by the factor which produced the increase.

(1) Including VAT of 20%, a radio costs £96. Find the original cost exclusive of VAT.

20% VAT added

$$100\% \xrightarrow{+20\%} 120\% = 1\cdot 20$$

$$\begin{aligned} £x \times 1\cdot 20 &= £96 \\ £x &= £96 \div 1\cdot 20 \\ &= £80 \end{aligned}$$

non-calculator: $120\% \xrightarrow{\div 12} 10\% \xrightarrow{\times 10} 100\%$

$£96 \div 12 = £8$ $\quad$ $£8 \times 10 = £80$

(2) A camera costs £120 after a discount of 25% is applied. Find the original cost.

25% discount subtracted

$$100\% \xrightarrow{-25\%} 075\% = 0\cdot 75$$

$$\begin{aligned} £x \times 0\cdot 75 &= £120 \\ £x &= £120 \div 0\cdot 75 \\ &= £160 \end{aligned}$$

non-calculator: $75\% = \frac{3}{4}$ $\quad$ $\frac{3}{4} \xrightarrow{\div 3} \frac{1}{4} \xrightarrow{\times 4} \frac{4}{4}$

$£120 \div 3 = £40$ $\quad$ $£40 \times 4 = £160$

COMPOUND PERCENTAGE CHANGE

appreciation and depreciation

(1) A £240000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15% in 2009. Calculate the value of the house at the end of 2009.

$$5\%\ \textit{increase}:\ 100\% + 5\% = 105\% = 1\cdot 05$$
$$10\%\ \textit{increase}:\ 100\% + 10\% = 110\% = 1\cdot 10$$
$$15\%\ \textit{decrease}:\ 100\% - 15\% = 085\% = 0\cdot 85$$

$$£240000 \times 1\cdot 05 \times 1\cdot 10 \times 0\cdot 85$$
$$= £235620$$

or

evaluate year by year

year 1

$$5\%\ \textit{of}\ £240000 = £12000$$
$$£240000 + £12000 = £25200$$

year 2

$$10\%\ \textit{of}\ £252000 = £25200$$
$$£252000 + £25200 = £277200$$

year 3

$$15\%\ \textit{of}\ £277200 = £41580$$
$$£277200 - £41580 = £235620$$

compound interest

(2) Calculate the compound interest on £12000 invested at 5% pa for 3 years.

$$£12000 \times (1\cdot 05)^3 \qquad \textit{ie.}\ \times 1\cdot 05 \times 1\cdot 05 \times 1\cdot 05$$

or evaluate year by year

$$£12000 \times 1\cdot 157625$$
$$= £13891\cdot 50$$
$$\textit{compound interest} = £13891\cdot 50 - £12000 = £1891\cdot 50$$

EXPRESSING CHANGE AS A PERCENTAGE

$$\%\ \textit{change} = \frac{\textit{change}}{\textit{start}} \times 100\%$$

A £15000 car is resold for £12000.
Find the percentage loss.

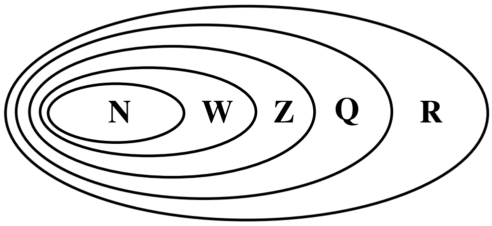
$$\textit{loss} = £15000 - £12000 = £3000$$

$$\%\ \textit{loss} = \frac{3000}{15000} \times 100\% = 20\%$$

CHAPTER 3: SURDS

NUMBER SETS:

Natural numbers N = {1, 2, 3...}
Whole numbers W = {0, 1, 2, 3...}
Integers Z = {... -3, -2, -1, 0, 1, 2, 3...}



Rational numbers, Q , can be written as a division of two integers.
Irrational numbers **cannot** be written as a division of two integers.

Real numbers, R , are all rational and irrational numbers.

SURDS ARE IRRATIONAL ROOTS.

For example, $\sqrt{2}$, $\sqrt{\frac{5}{9}}$, $\sqrt[3]{16}$ are surds.

whereas $\sqrt{25}$, $\sqrt{\frac{4}{9}}$, $\sqrt[3]{-8}$ are **not** surds as they are 5 , $\frac{2}{3}$ and -2 respectively.

SIMPLIFYING ROOTS:

RULES: $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$ $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$

(1) Simplify $\sqrt{24} \times \sqrt{3}$

$\sqrt{24} \times \sqrt{3}$
$= \sqrt{72}$

$= \sqrt{36} \times \sqrt{2}$ 36 *is the largest square number which is a factor of* 72

$= 6 \times \sqrt{2}$
$= 6\sqrt{2}$

(2) Simplify $\sqrt{72} + \sqrt{48} - \sqrt{50}$

$\sqrt{72} + \sqrt{48} - \sqrt{50}$
$= \sqrt{36} \times \sqrt{2} + \sqrt{16} \times \sqrt{3} - \sqrt{25} \times \sqrt{2}$
$= 6\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$
$= 6\sqrt{2} - 5\sqrt{2} + 4\sqrt{3}$
$= \sqrt{2} + 4\sqrt{3}$

(3) Remove the brackets and fully simplify:

(a) $\left(\sqrt{3}-\sqrt{2}\right)^2$

$= \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{3}\left(\sqrt{3}-\sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{9}-\sqrt{6} - \sqrt{6}+\sqrt{4}$

$= 3-\sqrt{6} - \sqrt{6}+2$

$= 5-2\sqrt{6}$

(b) $\left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= \left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= 3\sqrt{2}\left(3\sqrt{2}-2\right) + 2\left(3\sqrt{2}-2\right)$

$= 9\sqrt{4}-6\sqrt{2} + 6\sqrt{2} - 4$

$= 18-6\sqrt{2} + 6\sqrt{2} - 4$

$= 14$

RATIONALISING DENOMINATORS:
Removing surds from the denominator.

Express with a rational denominator:

(1) $\dfrac{4}{\sqrt{6}}$

$\dfrac{4}{\sqrt{6}}$

$= \dfrac{4 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}}$ *multiply the 'top' and 'bottom' by the surd on the denominator*

$= \dfrac{4\sqrt{6}}{6}$

$= \dfrac{2\sqrt{6}}{3}$

(2) $\dfrac{\sqrt{3}}{3\sqrt{2}}$

$\dfrac{\sqrt{3}}{3\sqrt{2}}$

$= \dfrac{\sqrt{3} \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}}$

$= \dfrac{\sqrt{6}}{3 \times \sqrt{4}}$

$= \dfrac{\sqrt{6}}{6}$

CHAPTER 4: INDICES

$$\text{base} \longrightarrow a^{n} \longleftarrow \text{index or exponent}$$

INDICES RULES: require the same base. **Examples:**

$$a^m \times a^n = a^{m+n} \qquad \frac{w^2 \times w^5}{w^3} = \frac{w^7}{w^3} = w^4$$

$$a^m \div a^n = a^{m-n}$$

$$\left(a^m\right)^n = a^{mn} \qquad \left(3^5\right)^2 = 3^{10}$$

$$(ab)^n = a^n b^n \qquad \left(2a^3b\right)^2 = 2^2 a^6 b^2 = 4a^6b^2$$

$$a^0 = 1 \qquad 5^0 = 1$$

$$a^1 = a \qquad 5^1 = 5$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad 8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$$

$$\frac{1}{a^p} = a^{-p} \qquad 8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}} = \frac{1}{16}$$

SCIENTIFIC NOTATION (STANDARD FORM)

Writing numbers in the form $a \times 10^n$

where $1 \le a < 10$ *place the decimal point after the first non-zero digit*
and n is an integer *numbers ...-3, -2, -1, 0, 1, 2, 3....*

$\mathbf{32800} = 3\cdot 28 \times 10 \times 10 \times 10 \times 10 = \mathbf{3 \cdot 28 \times 10^4}$

$\mathbf{0\cdot 000328} = 3\cdot 28 \div 10 \div 10 \div 10 \div 10 = 3\cdot 28 \div 10^4 = \mathbf{3\cdot 28 \times 10^{-4}}$

Notice for numbers starting $0\cdot$ the power of 10 is negative (same as $\div 10$).

SIGNIFICANT FIGURES indicate the accuracy of a **measurement**.

For example, 3400 cm = 34 m = 0·034 km
same measurement, same accuracy, each 2 significant figures.

Count the number of figures used, but **not** zeros at the **end** of a whole number
or zeros at the **start** of a decimal.

rounding: 5713·4 5700 to 2 significant figures
0·057134 0·057 to 2 significant figures (note: 0·057000 wrong)

(1) One **milligram** of hydrogen gas contans $2\cdot 987 \times 10^{20}$ molecules.
Calculate, to 3 significant figures, the number of molecules in **5 grams** of hydrogen.

$5000 \times 2\cdot 987 \times 10^{20}$ *learn to enter standard form in the calculator using the appropriate button* EE *or* EXP *or* x10^n *eg. 2·987* EXP *20*

$= 1\cdot 4935 \times 10^{24}$

$\approx 1\cdot 49 \times 10^{24}$ molecules

(2) The total mass of argon in a flask is $4\cdot 15 \times 10^{-2}$ grams.
The mass of a single atom of argon is $6\cdot 63 \times 10^{-23}$ grams.
Find, correct to 3 significant figures, the number of argon atoms in the flask.

$\dfrac{4\cdot 15 \times 10^{-2}}{6\cdot 63 \times 10^{-23}}$ *use the* (-) *button for a minus eg. 4·15* EXP (-) *2*

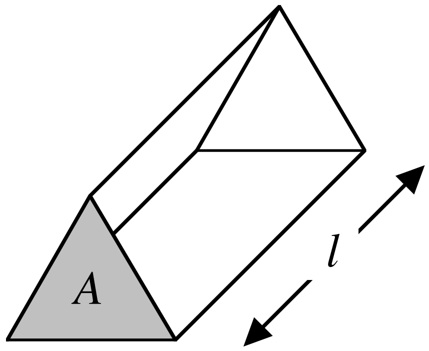
$= 6\cdot 259... \times 10^{20}$ *divide 'top' by 'bottom' and write the unrounded answer*

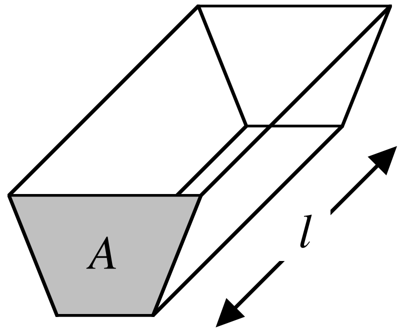
$\approx 6\cdot 26 \times 10^{20}$ atoms *write the rounded answer*

CHAPTER 5: VOLUMES OF SOLIDS

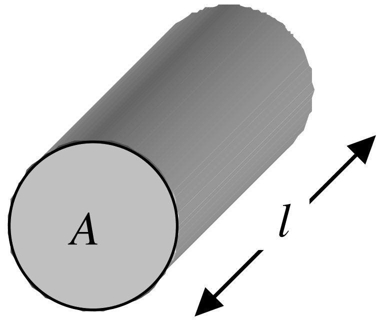
PRISMS a solid with the same cross-section throughout its length.
length l is at right-angles to the area A.

$V = Al$

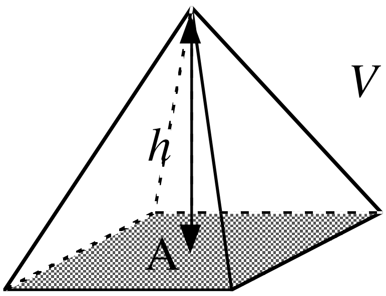




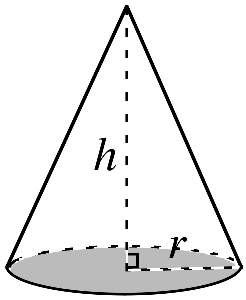
cylinder $V = \pi r^2 h$



PYRAMIDS



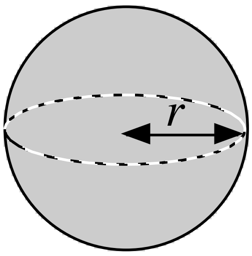
$V = \frac{1}{3} A h$



cone

$V = \frac{1}{3} \pi r^2 h$

SPHERE



$V = \frac{4}{3} \pi r^3$

EFFECT OF CHANGE

Describe the effect on the volume of a cylinder of:

(i) trebling the radius

$$V = \pi (3r)^2 h$$
$$= \pi 9 r^2 h$$
$$= 9\pi r^2 h$$

9 times bigger

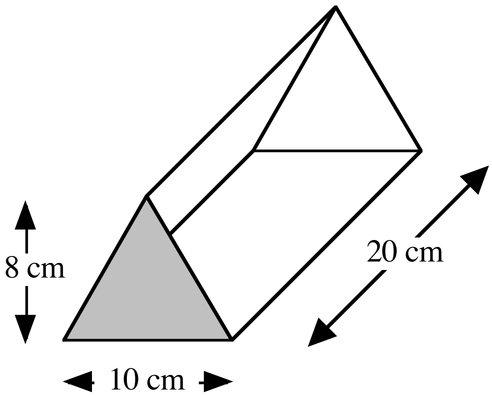
(ii) doubling the radius and halving the height

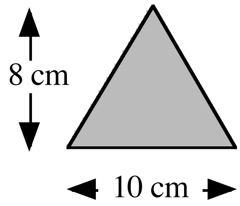
$$V = \pi (2r)^2 \left(\tfrac{1}{2} h\right)$$
$$= \pi 4 r^2 \times \tfrac{1}{2} h$$
$$= 2\pi r^2 h$$

2 times bigger

USING THE FORMULAE

(1) Calculate the volume.

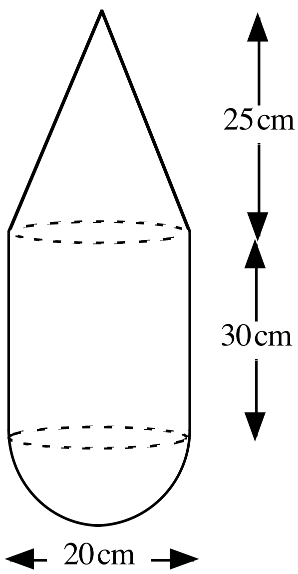




$$A = \tfrac{1}{2}bh$$
$$= 10 \times 8 \div 2$$
$$= 40\ cm^2$$

$$V = Al$$
$$= 40 \times 20$$
$$= 800\ cm^3$$

(2) Calculate the volume correct to **3 significant figures**.



$$radius = 20cm \div 2 = 10cm$$

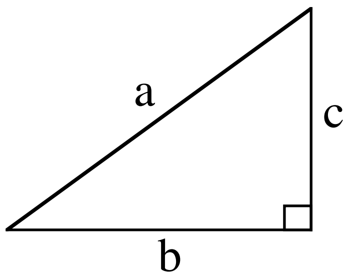
$$V = \tfrac{1}{3}\pi r^2 h$$
$$= \tfrac{1}{3} \times \pi \times 10 \times 10 \times 25$$
$$= 2617 \cdot 993\ldots\ cm^3$$

$$V = \pi r^2 h$$
$$= \pi \times 10 \times 10 \times 30$$
$$= 9424 \cdot 777\ldots\ cm^3$$

$$V = \tfrac{4}{3}\pi r^3 \div 2$$
$$= \tfrac{4}{3} \times \pi \times 10 \times 10 \times 10 \div 2$$
$$= 2094 \cdot 395\ldots\ cm^3$$

$$total\ volume = 2617 \cdot 993\ldots + 9424 \cdot 777\ldots + 2094 \cdot 395\ldots$$
$$= 14137 \cdot 166\ldots$$
$$\approx 14100\ cm^3$$

CHAPTER 6: PYTHAGORAS' THEOREM

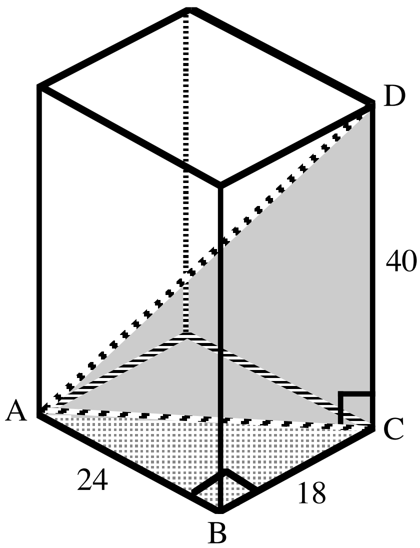


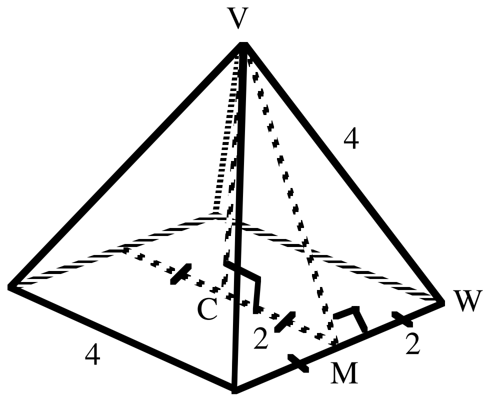
Side length **a** units is opposite the right angle.
It is the longest side, the **hypotenuse**.

For **right-angled triangles**:

$$a^2 = b^2 + c^2$$

3D Identify right-angled triangles in 3D diagrams.





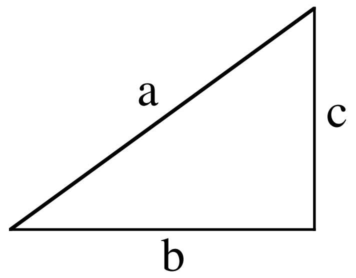
face diagonal
$$\begin{aligned} AC^2 &= 24^2 + 18^2 \\ &= 576 + 324 \\ &= 900 \\ AC &= \sqrt{900} \\ &= 30 \end{aligned}$$

slant height
$$\begin{aligned} VM^2 &= 4^2 - 2^2 \\ &= 16 - 4 \\ &= 12 \\ VM &= \sqrt{12} \\ &= 2\sqrt{3} \end{aligned}$$

space diagonal
$$\begin{aligned} AD^2 &= 40^2 + 30^2 \\ &= 1600 + 900 \\ &= 2500 \\ AD &= \sqrt{2500} \\ &= 50 \end{aligned}$$

height
$$\begin{aligned} VC^2 &= \left(\sqrt{12}\right)^2 - 2^2 \\ &= 12 - 4 \\ &= 8 \\ VC &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

CONVERSE OF PYTHAGORAS' THEOREM

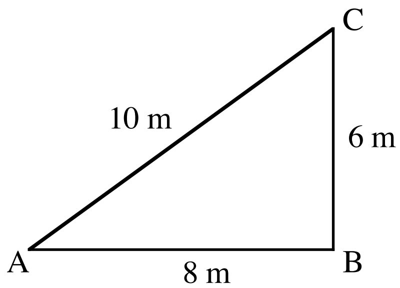


Longest side, length **a** units, is opposite the biggest angle.

If $a^2 = b^2 + c^2$

then the triangle is right-angled

Show that ΔABC is right angled.



$$AB^2 + BC^2 = 8^2 + 6^2 = 100$$
$$AC^2 = 10^2 = 100$$

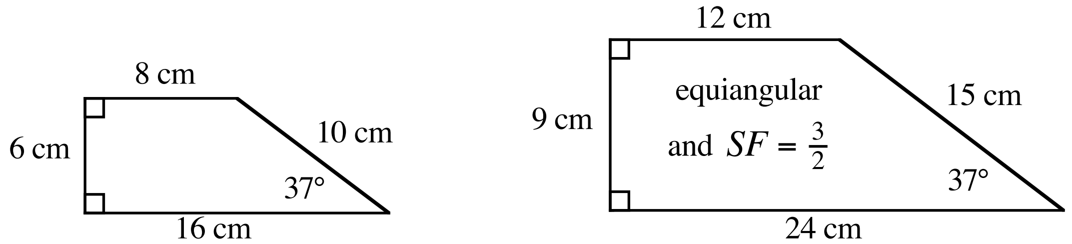
$$\textit{since } AB^2 + BC^2 = AC^2$$
$$\textit{by the Converse of Pyth. Thm.}$$
$$\Delta ABC \textit{ is right-angled at } B \quad (ie.\ \angle ABC = 90^\circ)$$

CHAPTER 7: SIMILAR SHAPES

Shapes are similar if they are enlargement or reductions of each other.

(1) the angles remain unchanged - the shapes are **equiangular**.

and (2) the sides are enlarged or reduced by some **scale factor** (SF).

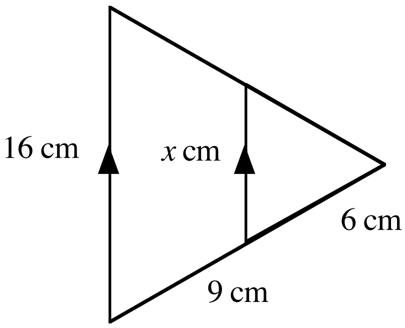


Triangles are special:
Enlarge or reduce sides by some scale factor and the two triangles will be equiangular.
If triangles are equiangular then they are similar.

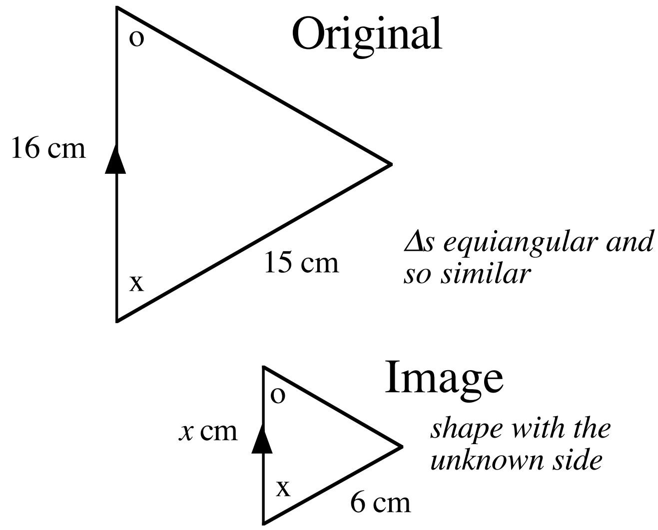
SCALING LENGTH

length scale factor, $\text{SF} = \dfrac{\text{image side}}{\text{original side}}$

enlargement if SF > 1
reduction if 0 < SF < 1



Find the value of *x*.

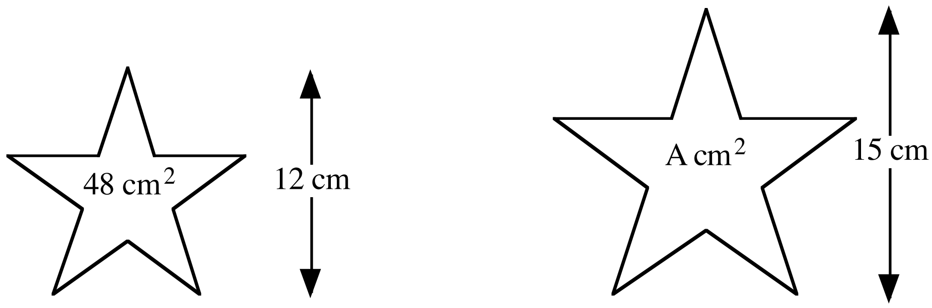


$$SF = \frac{image}{original} = \frac{6}{15} = \frac{2}{5}$$ *0 < SF < 1 as expected for a reduction*

$$x = \frac{2}{5} \times 16 = 6 \cdot 4$$ *smaller than 16 as expected for a reduction*

SCALING AREA for a 2D shape both length and breadth must be scaled.

length SF = n
area SF = n^2



Given that the two shapes shown are similar, find the area of the larger shape.

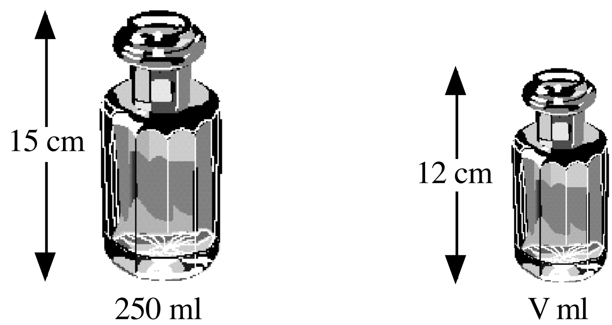
$$\textit{length}\ \text{SF} = \frac{\textit{image}}{\textit{original}} = \frac{15}{12} = \frac{5}{4}$$ *SF > 1 as expected for an enlargement*

$$\textit{area}\ \text{SF} = \frac{5}{4}\times\frac{5}{4} = \frac{25}{16}$$

$$A = \frac{25}{16}\times 48 = 75$$ *bigger than 48 as expected for an enlargement*

SCALING VOLUME for a 3D shape length, breadth and height must be scaled.

length SF = n
volume SF = n^3



Given that the two solids shown are similar find the volume of the smaller solid.

$$\textit{length}\ \text{SF} = \frac{\textit{image}}{\textit{original}} = \frac{12}{15} = \frac{4}{5}$$ *0 < SF < 1 as expected for a reduction*

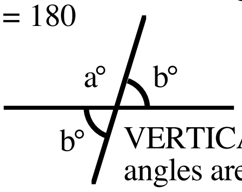
$$\textit{volume}\ \text{SF} = \frac{4}{5}\times\frac{4}{5}\times\frac{4}{5} = \frac{64}{125}$$

$$V = \frac{64}{125}\times 250 = 128$$ *smaller than 250 as expected for a reduction*

CHAPTER 8: ANGLES and SHAPE

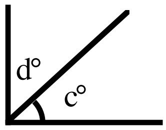
LINES

SUPPLEMENTARY angles
a + b = 180

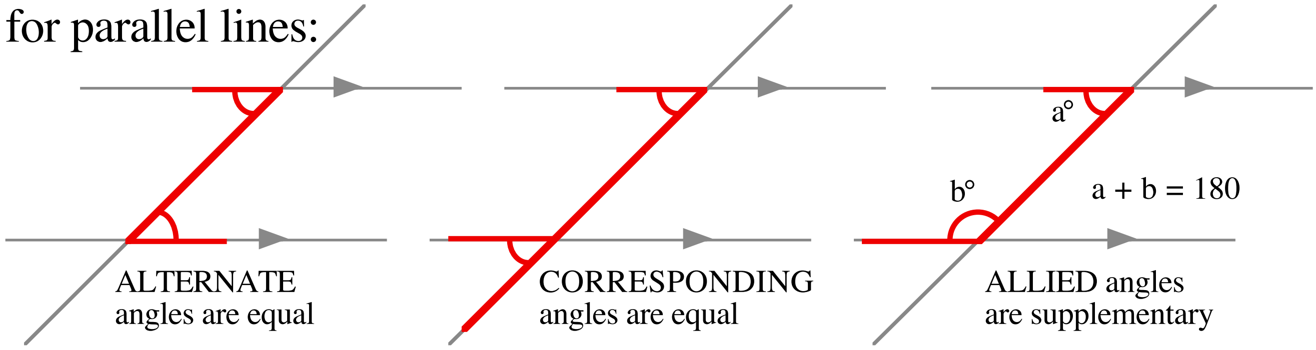


VERTICALLY OPPOSITE
angles are equal

COMPLEMENTARY angles
c + d = 90



for parallel lines:



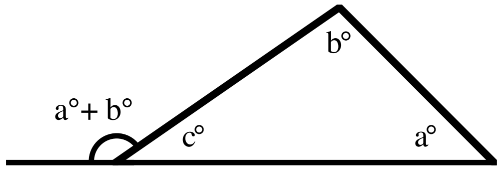
ALTERNATE
angles are equal

CORRESPONDING
angles are equal

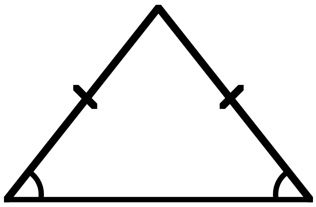
ALLIED angles
are supplementary

TRIANGLES

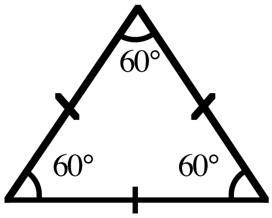
angle sum a°+ b°+ c° = 180°



EXTERIOR ANGLE is the sum of the opposite INTERIOR ANGLES



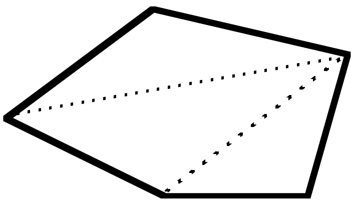
ISOSCELES



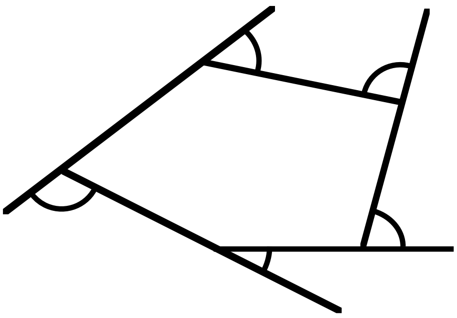
EQUILATERAL

POLYGONS

INTERIOR ANGLES sum to (n-2)×180° where n sides
EXTERIOR ANGLES sum to 360°

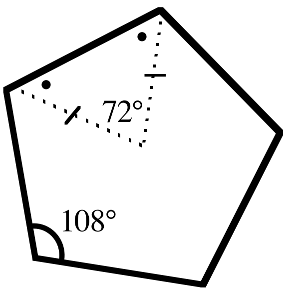


pentagon
interior sum 3×180° = 540°



exterior sum 360°

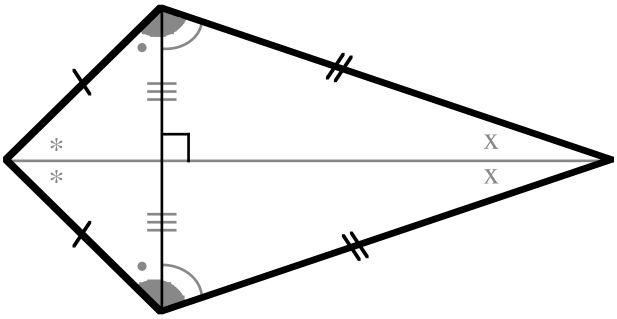
all sides and angles equal

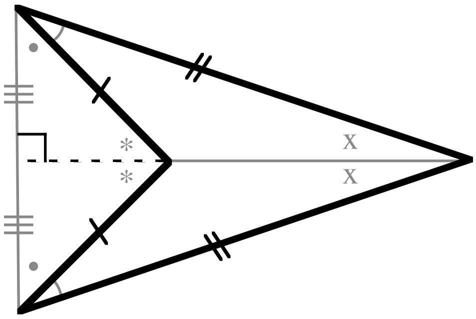


REGULAR pentagon
interior angle 540°÷5 = 108°

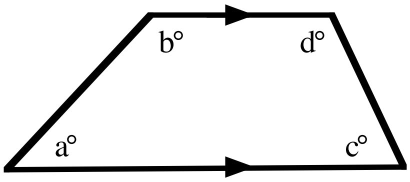
QUADRILATERALS

KITE

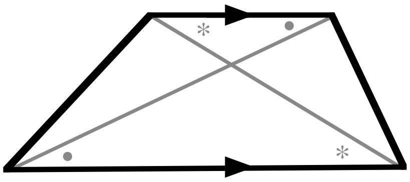




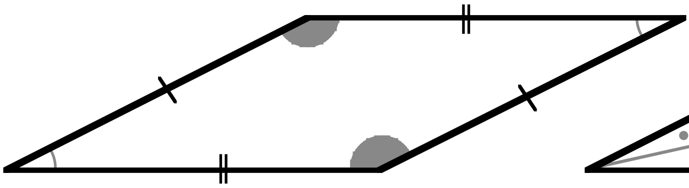
TRAPEZIUM

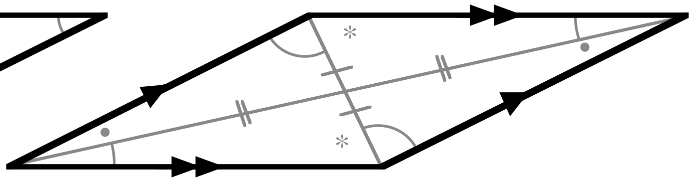


a + b = 180 , c + d = 180

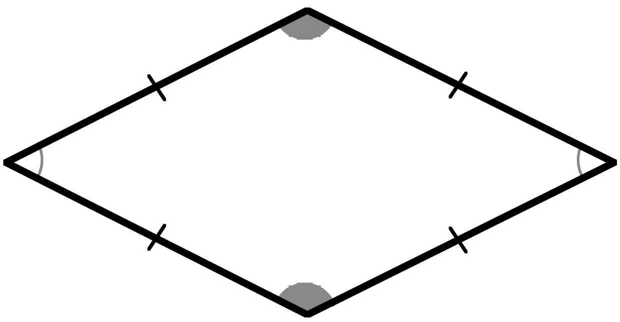


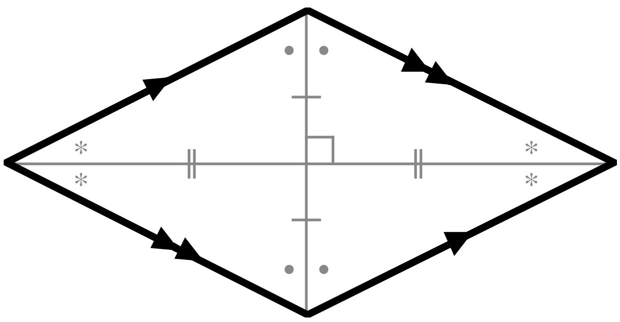
PARALLELOGRAM a trapezium



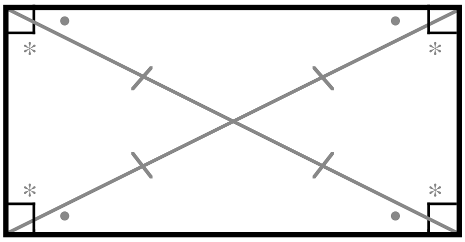


RHOMBUS a kite and parallelogram

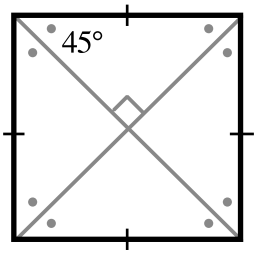




RECTANGLE a parallelogram

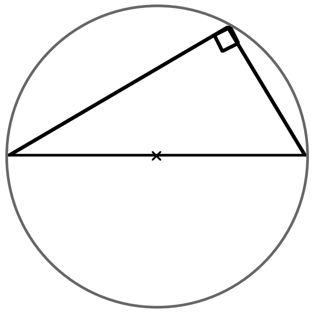


SQUARE a rectangle and rhombus.

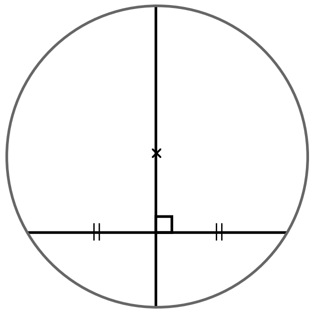


CHAPTER 9: PROPERTIES OF THE CIRCLE

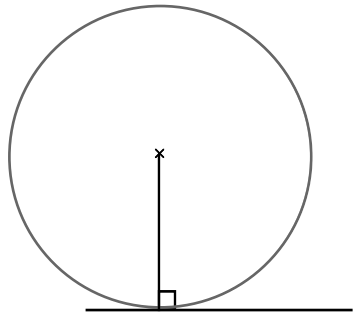
angle in a semicircle is a right-angle.



the perpendicular bisector of a chord is a diameter.

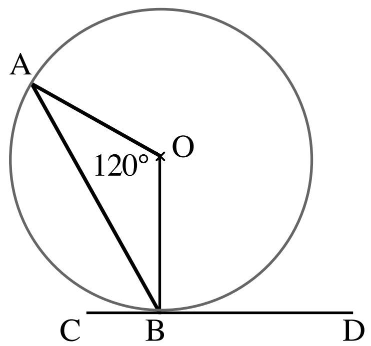


a tangent and the radius drawn to the point of contact form a right-angle.



ANGLES

(1)



radius OA = OB so ΔAOB is isosceles and Δ angle sum 180° :

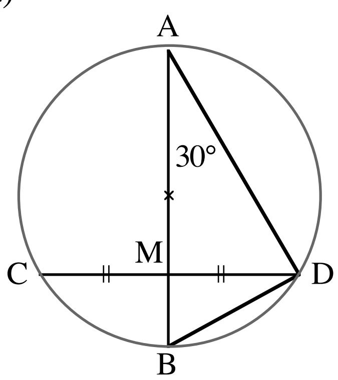
$\angle OBA = (180° - 120°) \div 2 = 30°$

tangent CD and radius OB : $\angle OBC = 90°$

Calculate the size of angle ABC.

$\angle ABC = 90° - 30° = 60°$

(2)



diameter AB bisects chord CD : $\angle AMD = 90°$

ΔAMD *angle sum* $180°$:

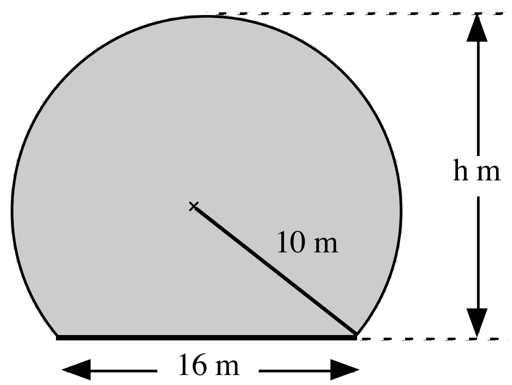
$\angle ADM = 180° - 90° - 30° = 60°$

angle in a semicircle : $\angle ADB = 90°$

Calculate the size of angle BDC.

$\angle BDC = 90° - 60° = 30°$

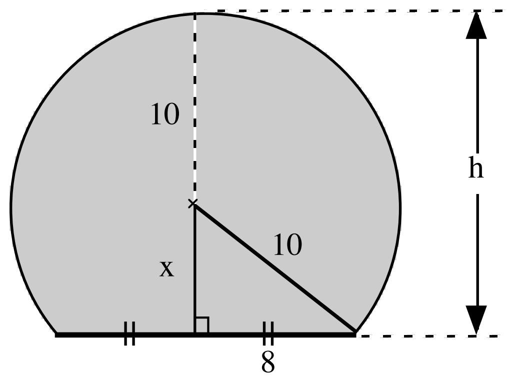
PYTHAGORAS' THEOREM



A circular road tunnel, radius 10 metres, is cut through a hill.

The road has a width 16 metres.

Find the height of the tunnel.



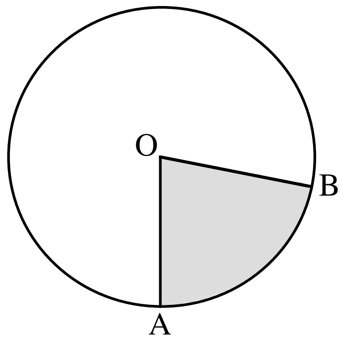
the diameter drawn is the perpendicular bisector of the chord:
Δ is right-angled so can apply Pyth. Thm.

$$\begin{aligned} x^2 &= 10^2 - 8^2 \\ &= 100 - 64 \\ &= 36 \\ x &= \sqrt{36} \\ x &= 6 \end{aligned}$$

$$\begin{aligned} h &= x + 10 \\ &= 6 + 10 \\ h &= 16 \end{aligned}$$

height 16 *metres*

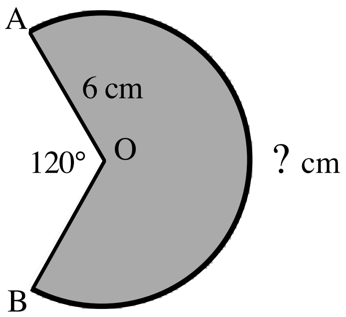
CHAPTER 10: ARCS and SECTORS



$$\frac{\angle AOB}{360^\circ} = \frac{arc\ AB}{\pi d} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$

Choose the appropriate pair of ratios based on:
(i) the ratio which includes the quantity to be found
(ii) the ratio for which both quantities are known (or can be found).

(1) Find the **exact** length of **major** arc AB.

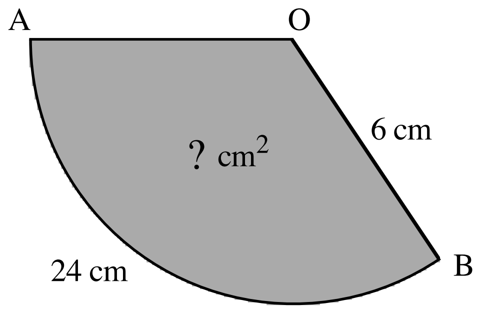


$$diameter\ d = 2\times 6\ cm = 12\ cm$$
$$reflex\ \angle AOB = 360^\circ - 120^\circ = 240^\circ$$

$$\frac{\angle AOB}{360^\circ} = \frac{arc\ AB}{\pi d}$$
$$\frac{240^\circ}{360^\circ} = \frac{arc\ AB}{\pi \times 12}$$

$$arc\ AB = \frac{240^\circ}{360^\circ}\times \pi \times 12$$
$$= 8\pi\ cm \quad (25\cdot 132...)$$

(2) Find the **exact** area of sector AOB.



$$\frac{arc\ AB}{\pi d} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$
$$\frac{24}{\pi\times 12} = \frac{area\ of\ sector\ AOB}{\pi\times 6\times 6}$$

$$area\ of\ sector\ AOB = \frac{24}{\pi\times 12}\times\pi\times 6\times 6$$
$$= 72\ cm^2$$

CHAPTER 11: ALGEBRAIC EXPRESSIONS

REMOVING BRACKETS

SINGLE BRACKETS

(1) $3x\,(2x - y + 7)$

$= 6x^2 - 3xy\ + 21x$

$3x \times 2x = 6x^2$
$3x \times -y = -3xy$
$3x \times +7 = +21x$

(2) $-2\,(3t + 5)$

$= -6t - 10$

$-2 \times 3t = -6t$
$-2 \times +5 = -10$

(3) $-3w\,(w^2 - 4)$

$= -3w^3 + 12w$

$-3w \times w^2 = -3w^3$
$-3w \times -4 = +12w$

Fully simplify:

(4) $2t\,(3 - t) + 5t^2$

$= 6t - 2t^2 \quad + 5t^2$

$= 6t\ + 3t^2$

(5) $5 - 3\,(n - 2)$

$= 5 - 3n + 6$

$= 5 + 6 - 3n$

$= 11 - 3n$

DOUBLE BRACKETS

(1) $(3x + 2)(2x - 5)$

$= 3x\,(2x - 5)\ +\ 2\,(2x - 5)$

$= 6x^2 - 15x \quad + \quad 4x - 10$

$= 6x^2 - 11x - 10$

or

"FOIL"

$(3x + 2)(2x - 5)$

(2) $(2t - 3)^2$

$= (2t - 3)(2t - 3)$

$= 2t\,(2t - 3)\ -\ 3\,(2t - 3)$

$= 4t^2 - 6t \quad - \quad 6t + 9$

$= 4t^2 - 12t\ + 9$

(3) $(w + 2)(w^2 - 3w + 5)$

$= w\,(w^2 - 3w + 5)\ +\ 2\,(w^2 - 3w + 5)$

$= w^3 - 3w^2 + 5w \quad + \quad 2w^2 - 6w + 10$

$= w^3 - 3w^2 + 2w^2 + 5w - 6w + 10$

$= w^3 - w^2 - w + 10$

FACTORSATION

COMMON FACTORS $\quad ab + ac = a(b + c)$

Highest Common Factors are used to write expressions in **fully** factorised form.

Factorise **fully**: $\quad 4a - 2a^2$

$= 2a(2 - a) \qquad 2a \times 2 - 2a \times a \;\; using \;\; HCF(4a, 2a^2) = 2a$

NOTE: the following answers are factorised but not **fully** factorised:

$2(2a - a^2)$

$a(4 - 2a)$

DIFFERENCE OF TWO SQUARES $\quad a^2 - b^2 = (a + b)(a - b)$

Factorise **fully**:

(1) $4x^2 - 9$

$= (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$

(2) $4x^2 - 36$

$= 4(x^2 - 9)$ *common factor first*

$= 4(x + 3)(x - 3)$

NOTE: $(2x + 6)(2x - 6)$ is factorised but not **fully** factorised.

TRINOMIALS

$$ax^2 + bx + c \;, a = 1 \quad \text{ie. } 1x^2$$

$x^2 + bx + c = (x + ?)(x + ?)$ The missing numbers are: **a pair of factors of** c that **sum to** b

Factorise **fully**:

(1) $x^2 + 5x + 6$	(2) $x^2 - 5x + 6$	(3) $x^2 - 5x - 6$
$1 \times 6 = 2 \times 3 = 6$	$-1, -6 \text{ or } -2, -3$	$-1,6$ or $1,-6$ or $-2,3$ or $2,-3$
$2 + 3 = 5$	$-2 + (-3) = -5$	$1 + (-6) = -5$
use $+2$ *and* $+3$	*use* -2 *and* -3	*use* $+1$ *and* -6
$= (x + 2)(x + 3)$	$= (x - 2)(x - 3)$	$= (x + 1)(x - 6)$

$$ax^2 + bx + c \;, a \neq 1 \quad \text{ie. not } 1x^2$$

Try out the possible combinations of the factors which could be in the brackets.

Factorise $3t^2 - 10t - 8$

$3 \times (-8) = -24$ *pairs of factors* $\underbrace{1,24 \text{ or } 2,12 \text{ or } 3,8 \text{ or } 4,6}_{-12 + 2 = -10}$ *one factor is negative*

$3t^2 \;\underline{\underline{\mathbf{-10t}}} - 8$

factors sum to -10 $\qquad -12t + 2t = -10t$

try combinations so that $-12t$ *and* $2t$ *are obtained*

factor pairs: $3t \times t$ *for* $3t^2$ $\qquad 1 \times 8$ *or* 2×4 *for* 8, *one factor negative*

$3t$	$+1$	$3t$	-8	$3t$	$+2$	$3t$	$+4$
t	-8	t	$+1$	t	-4	t	-2
$1t$	$-24t$	$-8t$	$3t$	$2t$	$-12t$ ✓	$4t$	$-6t$

$$\underline{\underline{(3t + 2)(t - 4)}}$$

ALGEBRAIC FRACTIONS

SIMPLIFYING: fully factorise 'top' and 'bottom' and 'cancel' common factors

MULTIPLY/DIVIDE:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \qquad\qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

ADD/SUBTRACT: a common denominator is required.

(1) $\dfrac{x^2 - 9}{x^2 + 2x - 3}$

$= \dfrac{(x-3)(x+3)}{(x-1)(x+3)}$ *fully factorise*

$= \dfrac{x-3}{x-1}$ *cancel common factors*

(2) $\dfrac{4}{ab} \div \dfrac{8a}{b^2}$

$= \dfrac{4}{ab} \times \dfrac{b^2}{8a}$

$= \dfrac{4 \times b^2}{ab \times 8a}$

$= \dfrac{4 \times b \times b}{8 \times a \times a \times b}$

$= \dfrac{1 \times b}{2 \times a \times a}$

$= \dfrac{b}{2a^2}$

(3) $\dfrac{x}{2} - \dfrac{x-3}{3}$

$= \dfrac{3x}{6} - \dfrac{2(x-3)}{6}$

$= \dfrac{3x - 2(x-3)}{6}$

$= \dfrac{3x - 2x + 6}{6}$ *notice sign change*

$= \dfrac{x+6}{6}$

(4) $\dfrac{1}{x} + \dfrac{3}{x(x-3)}$

$= \dfrac{1(x-3)}{x(x-3)} + \dfrac{3}{x(x-3)}$

$= \dfrac{x-3+3}{x(x-3)}$

$= \dfrac{x}{x(x-3)}$ *can simplify*

$= \dfrac{1}{x-3}$

CHAPTER 12: EQUATIONS and INEQUALITIES

Simplify by following the rules:

addition and subtraction

$$x + a = b \qquad x - a = b$$
$$x = b - a \qquad x = b + a$$

multiplication and division

$$ax = b \qquad \frac{x}{a} = b$$
$$x = \frac{b}{a} \qquad x = ab$$

(1) Solve: $5x - 4 = 2x - 19$

$3x - 4 = -19$ *subtracted $2x$ from each side*

$3x = -15$ *added 4 to each side*

$x = -5$ *divided each side by 3*

WITH BRACKETS remove first and simplify

(2) Solve: $(4x + 3)(x - 2) = (2x - 3)^2$

$4x^2 - 5x - 6 = 4x^2 - 12x + 9$ *removed brackets, fully simplifying*

$-5x - 6 = -12x + 9$ *subtracted $4x^2$ from each side*

$7x - 6 = +9$ *added $12x$ to each side*

$7x = 15$ *added 6 to each side*

$x = \frac{15}{7}$ *divided each side by 7*

WITH FRACTIONS remove first, multiplying by the LCM of the denominators

(3) Solve $\frac{1}{2}(x + 3) + \frac{1}{3}x = 1$

$\frac{3}{6}(x + 3) + \frac{2}{6}x = \frac{6}{6}$ *write with common denominators*

$3(x + 3) + 2x = 6$ *both sides $\times 6$ to remove fractions*

$3x + 9 + 2x = 6$

$5x = -3$

$x = -\frac{3}{5}$

INEQUALITIES

Follow the rules for equations, except:

multiply or divide by a **negative** number, reverse the direction of the inequality sign

$-ax > b \qquad \dfrac{x}{-a} > b$

$x < \dfrac{b}{-a} \qquad x < -ab$

(1) $8 + 3x > 2$

$+3x > -6$

$x > \dfrac{-6}{+3}$ *divided each side by* $+3$ *notice sign unchanged*

$x > -2$

(2) $8 - 3x > 2$

$-3x > -6$ *subtracted* 8 *from each side*

$x < \dfrac{-6}{-3}$ *divided each side by* -3 *notice sign reversed*

$x < 2$ *simplified*

RESTRICTIONS ON SOLUTIONS

(1) $x \le \dfrac{5}{2}$ *where x is a whole number*

$x = 0,1,2$

(2) $-2 \le x < 2$ *where x is an integer*

$x = -2,-1,0,1$

TRANSPOSING FORMULAE (CHANGE OF SUBJECT)

Follow the rules for equations to isolate the **target letter**.

addition and subtraction

$$x + a = b \qquad x - a = b$$

$$x = b - a \qquad x = b + a$$

multiplication and division

$$ax = b \qquad \frac{x}{a} = b$$

$$x = \frac{b}{a} \qquad x = ab$$

powers and roots

$$x^2 = a \qquad \sqrt{x} = a$$

$$x = \sqrt{a} \qquad x = a^2$$

$F = 3r^2 + p$ Change the subject of the formula to r.

$$r \xrightarrow{\text{square}} r^2 \xrightarrow{\times 3} 3r^2 \xrightarrow{+p} F$$

$$\sqrt{\frac{F-p}{3}} \xleftarrow{\sqrt{\;}} \frac{F-p}{3} \xleftarrow{\div 3} F - p \xleftarrow{-p} F$$

inverse operations in reverse order

$F = 3r^2 + p$

$F - p = 3r^2$ *subtract p from each side*

$\frac{F-p}{3} = r^2$ *divide each side by 3*

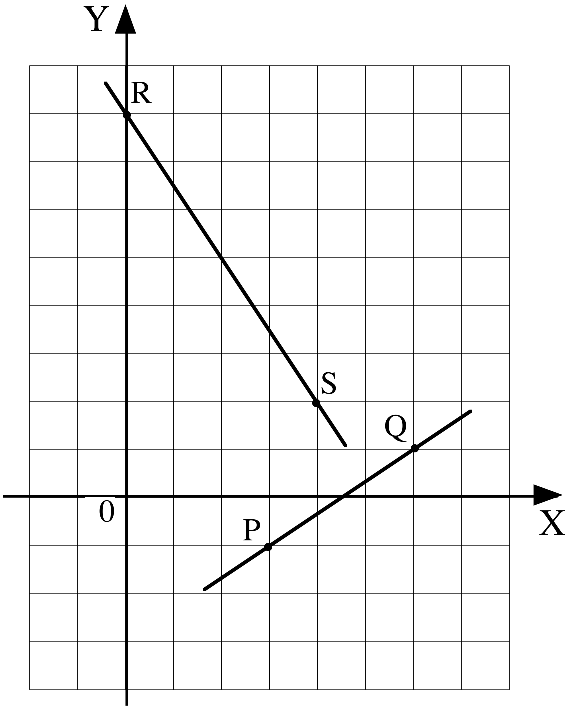
$\sqrt{\frac{F-p}{3}} = r$ *square root both sides*

$r = \sqrt{\frac{F-p}{3}}$ *subject of formula now r*

CHAPTER 13: STRAIGHT LINE

GRADIENT The slope of a line is given by the ratio: $$m = \frac{\text{vertical change}}{\text{horizontal change}}$$

Using coordinates, the **gradient formula** is $$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$



$R\,(0,8)$, $S\,(4,2)$

$$m_{RS} = \frac{y_S - y_R}{x_S - x_R} = \frac{2-8}{4-0} = \frac{-6}{4} = -\frac{3}{2}$$

$P\,(3,-1)$, $Q\,(6,1)$

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{1-(-1)}{6-3} = \frac{2}{3}$$

note: same result for $$\frac{-1-1}{3-6} = \frac{-2}{-3} = \frac{2}{3}$$

EQUATION OF A STRAIGHT LINE

gradient m , y-intercept C units ie. meets the y-axis at $(0,C)$ $$y = mx + C$$

gradient m , through the point (a,b) $$y - b = m(x - a)$$

equation of line RS:

$m_{RS} = -\frac{3}{2}$ $\qquad y = mx + C$

$R(0,8)\quad C = 8$ $\qquad y = -\frac{3}{2}x + 8$

equation of line PQ:

$m_{PQ} = \frac{2}{3}$ $\qquad y - b = m(x - a)$

$\;\;a\;\;\;b$

$Q(\,6\;,1\,)$ $\qquad y - 1 = \frac{2}{3}(x - 6)$

or use point P

$$3y - 3 = 2(x - 6)$$

$$3y - 3 = 2x - 12$$

$$3y = 2x - 9$$

Rearrange the equation to $y = mx + C$ for the gradient and y-intercept.

$$3x + 2y - 16 = 0$$

$$2y = -3x + 16 \qquad \textit{isolate y - term}$$

$$y = -\frac{3}{2}x + 8 \qquad \textit{obtain } 1y =$$

$$y = mx + C \qquad \textit{compare to the general equation}$$

$$m = -\tfrac{3}{2} \ , \quad C = 8 \qquad \textit{meets the Y - axis at } (0,8)$$

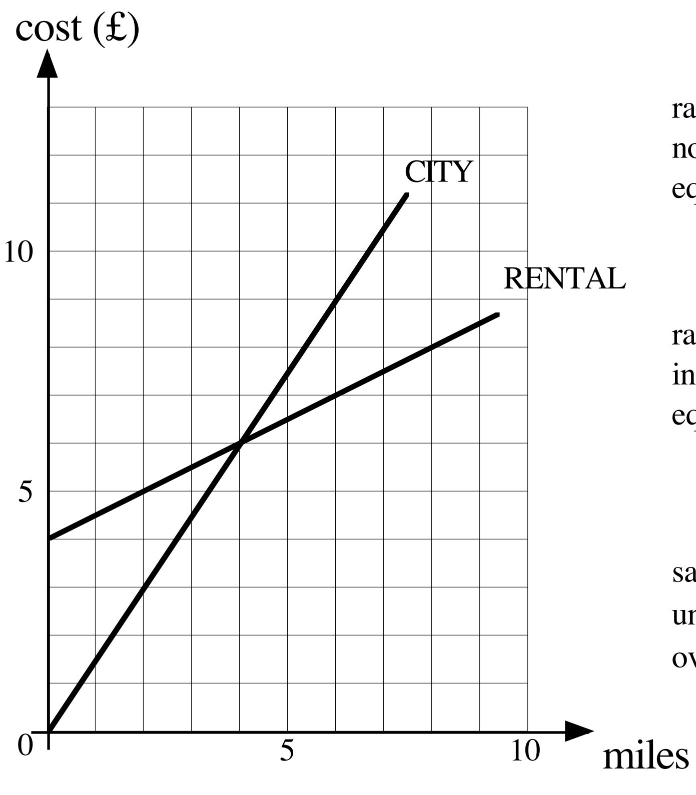
RATE OF CHANGE

The gradient is the rate of change.

distance/time graphs: the gradient is the speed.
speed/time graphs: the gradient is the acceleration.

The steeper the graph, the greater the rate of change.
A positive gradient is an increase, a negative gradient is a decrease.
Gradient zero (horizontal line), no change.

RENTAL TAXIS requires a £4 payment plus a charge of 50p per mile.
CITY TAXIS charges £1.50 per mile.
Advise on the better buy.

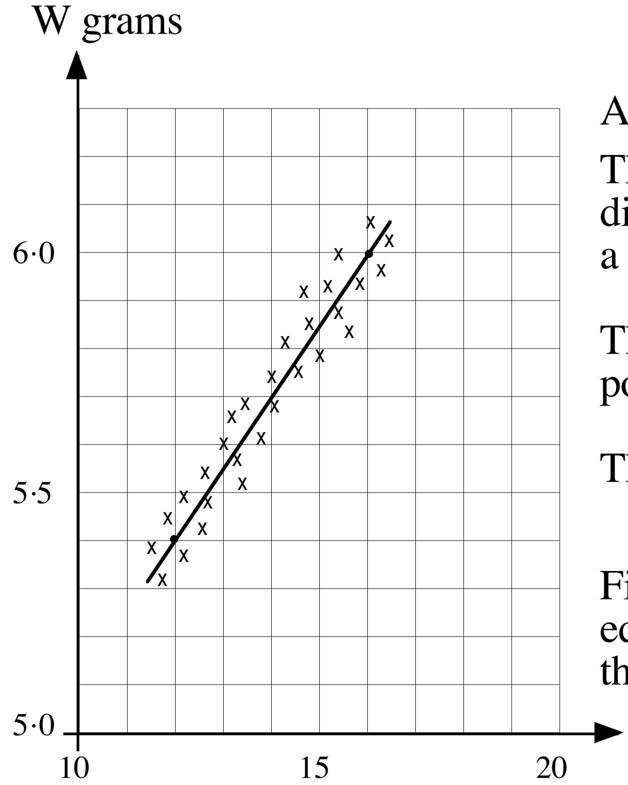


rate of change £1·50 per mile , no initial charge , equation $y = 1{\cdot}5x$ — m = 1·5, C = 0

rate of change £0·50 per mile , initial charge £4 , equation $y = 0{\cdot}5x + 4$ — m = 0·5, C = 4

same cost for 4 miles, but
under 4 miles CITY cheaper
over 4 miles RENTAL cheaper

SCATTER DIAGRAM: LINE OF BEST FIT



A salt is dissolved in a litre of solvent.

The amount of salt that dissolves at different temperatures is recorded and a graph plotted.

The best-fitting straight line through the points is drawn.

The equation of the graph is of the form

$$W = mT + C.$$

Find the equation of the line and use the equation to calculate the mass of salt that will dissolve at 30 °C.

using two well-separated points on the line $(16\ ,\ 6\cdot 0)$ $(12\ ,\ 5\cdot 4)$

$$m = \frac{6\cdot 0 - 5\cdot 4}{16-12} = \frac{0\cdot 6}{4} = 0\cdot 15$$

substituting for one point on the line $(\overset{a}{16}\ ,\ \overset{b}{6\cdot 0})$

$$y - b = m(x - a)$$

$$y - 6\cdot 0 = 0\cdot 15(x - 16)$$

$$y - 6\cdot 0 = 0\cdot 15\,x - 2\cdot 4$$

$$y = 0\cdot 15\,x + 3\cdot 6$$

$$\underline{\underline{W = 0\cdot 15T + 3\cdot 6}}$$

$T = 30$

$$\begin{aligned} W &= 0\cdot 15 \times 30 + 3\cdot 6 \\ &= 4\cdot 5 + 3\cdot 6 \\ &= 8\cdot 1 \end{aligned}$$

$\underline{\underline{8\cdot 1\ \textit{grams}}}$

CHAPTER 14: SIMULTANEOUS EQUATIONS

The equation of a line is a rule connecting the x and y coordinates of any point on the line.

For example, $\begin{matrix} x & y \\ (-2, & 12) \end{matrix}$ $\qquad \begin{aligned} y + 2x &= 8 \\ 12 + 2 \times (-2) &= 8 \end{aligned}$

To sketch a line, find the points where the line meets the axes.

SOLVE SIMULTANEOUS EQUATIONS: GRAPHICAL METHOD

Sketch the two lines and the point of intersection is the solution.

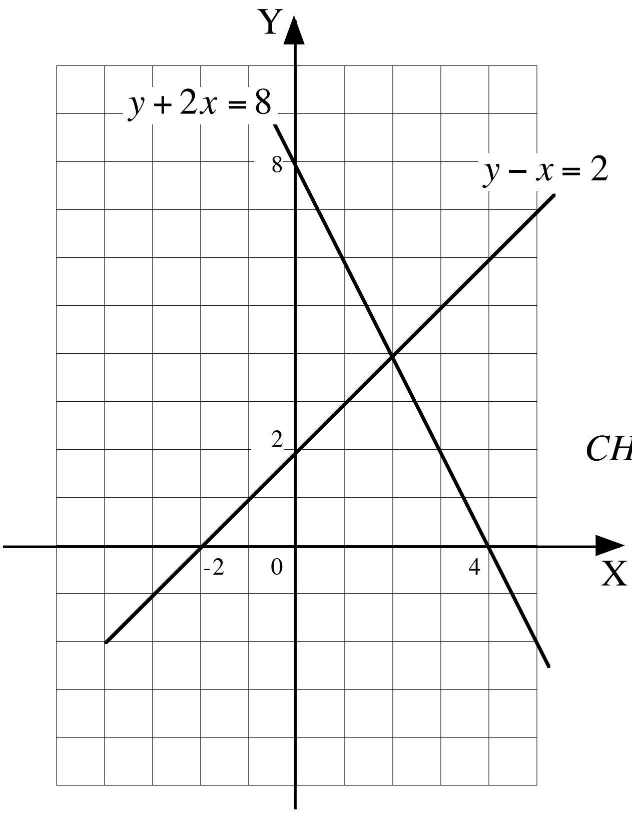
Solve **graphically** the system of equations: $\begin{aligned} y + 2x &= 8 \\ y - x &= 2 \end{aligned}$

$$\begin{aligned} y + 2x &= 8 && (1) \\ y + 2 \times 0 &= 8 && \textit{substituted for } x = 0 \\ y &= 8 && \textit{plot } (0,8) \end{aligned}$$

$$\begin{aligned} y - x &= 2 && (2) \\ y - 0 &= 2 && \textit{substituted for } x = 0 \\ y &= 2 && \textit{plot } (0,2) \end{aligned}$$

$$\begin{aligned} y + 2x &= 8 && (1) \\ 0 + 2x &= 8 && \textit{substituted for } y = 0 \\ x &= 4 && \textit{plot } (4,0) \end{aligned}$$

$$\begin{aligned} y - x &= 2 && (2) \\ 0 - x &= 2 && \textit{substituted for } y = 0 \\ \mathrm{x} &= -2 && \textit{plot } (-2,0) \end{aligned}$$



point of intersection $(2,4)$

SOLUTION: $x = 2$ *and* $y = 4$

CHECK: $x = 2$ *and* $y = 4$ *substituted in both equations*

$$\begin{aligned} y + 2x &= 8 && (1) \\ 4 + 2 \times 2 &= 8 \\ 8 &= 8 \end{aligned}$$

$$\begin{aligned} y - x &= 2 && (2) \\ 4 - 2 &= 2 \\ 2 &= 2 \end{aligned}$$

SOLVE SIMULTANEOUS EQUATIONS: SUBSTITUTION METHOD

Rearrange one equation to $y =$ and substitute for y in the other equations
(or $x =$)

Solve **algebraically** the system of equations: $y + 2x = 8$
$3y - x = 10$

$y + 2x = 8$ (1) *can choose to rearrange to* $y =$ *or* $x =$

$y = 8 - 2x$ *choosing* $y =$ *avoids fractions as* $x = 4 - \frac{1}{2}y$

$3y - x = 10$ (2)

$3(8 - 2x) - x = 10$ *replace y by* $8 - 2x$

$24 - 6x - x = 10$ *solve*

$-7x = -14$

$x = 2$

$y = 8 - 2x$ (1) *can choose either equation* (1) *or* (2)

$= 8 - 2 \times 2$ *substituted for* $x = 2$

$y = 4$

SOLUTION: $x = 2$ *and* $y = 4$

CHECK:

$3y - x = 10$ (2) *using the other equation*

$3 \times 4 - 2 = 10$ *substituted for* $x = 2$ *and* $y = 4$

$10 = 10$

SOLVE SIMULTANEOUS EQUATIONS: ELIMINATION METHOD

Can add or subtract multiples of the equations to eliminate either the x or y term.

Solve **algebraically** the system of equations: $4x + 3y = 5$

$$5x - 2y = 12$$

$4x + 3y = 5$	$(1) \times 2$	*can choose to eliminate x or y term*
$5x - 2y = 12$	$(2) \times 3$	*choosing y term*, $LCM\,(3y, 2y) = 6y$
$8x + 6y = 10$	(3)	*multiplied each term of* (1) *by* 2 *for* $+6y$
$15x - 6y = 36$	(4)	*multiplied each term of* (2) *by* 3 *for* $-6y$
$23x \quad = 46$	$(3) + (4)$	*added equations, adding "like" terms*
$x = 2$		$+6y$ *and* $-6y$ *added to* 0 (*ie eliminate*)
$4x + 3y = 5$	(1)	*can choose either equation* (1) *or* (2)
$4 \times 2 + 3y = 5$		*substituted for* $x = 2$
$8 + 3y = 5$		
$3y = -3$		
$y = -1$		

SOLUTION: $x = 2$ *and* $y = -1$

CHECK:

$5x - 2y = 12$	(2)	*using the other equation*
$5 \times 2 - 2 \times (-1) = 12$		*substituted for* $x = 2$ *and* $y = -1$
$10 - (-2) = 12$		
$12 = 12$		

CHAPTER 15: QUADRATIC EQUATIONS

A function pairs one number with another, its IMAGE. It can be defined by a formula.

QUADRATIC FUNCTIONS

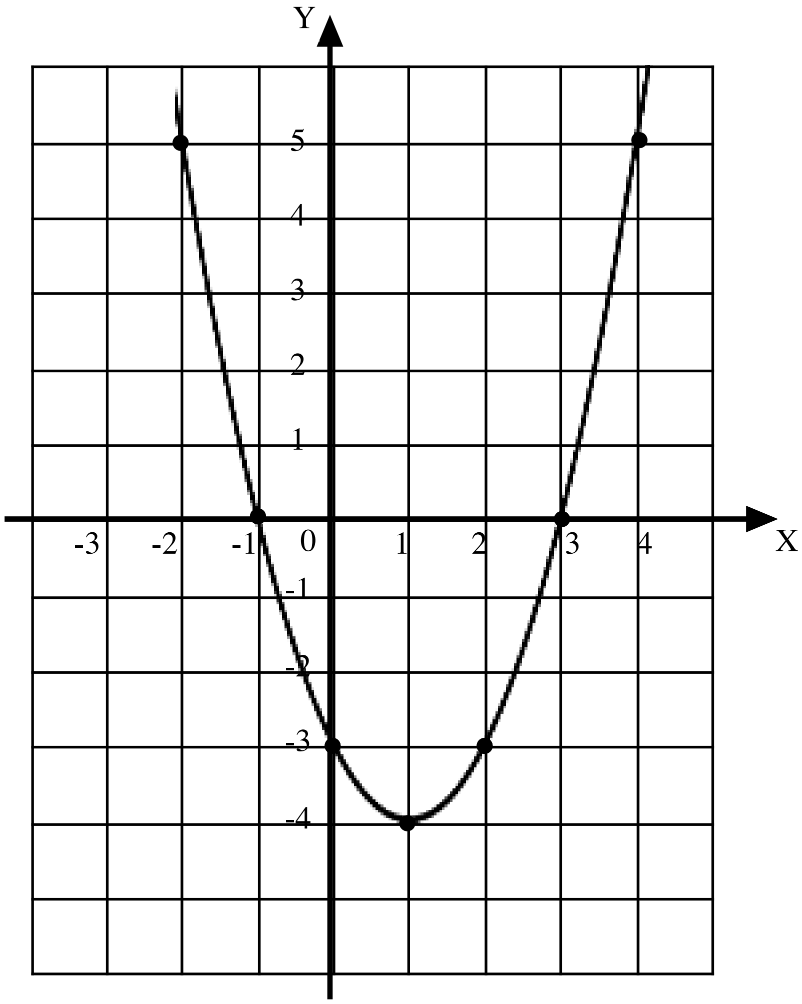
$f(x) = ax^2 + bx + c$, $a \neq 0$, where a, b and c are constants.

$f(x) = x^2 - 2x - 3$

$f(-2) = (-2)^2 - 2 \times (-2) - 3 \ = \ 4 + 4 - 3 \ = \ 5$, the image of -2 is 5.

x	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5
points	(-2,5)	(-1,0)	(0,-3)	(1,-4)	(2,-3)	(3,0)	(4,5)

If all possible values of x are considered, a graph will show the images (the RANGE).
The graph is a symmetrical curve called a PARABOLA.



The graph meets the x-axis where $x^2 - 2x - 3 = 0$.
The **zeros** of the graph are -1 and 3 which are the **roots** of the equation.

QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0,\ a \neq 0$, where a, b and c are constants.

The values of x that satisfy the equation are the **roots** of the equation.
The quadratic formula can be used to solve the equation.

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , a \neq 0$$

Find the **roots** of the equation $3x^2 - 5x - 1 = 0$, correct to two decimal places.

$3x^2 - 5x - 1 = 0$
$ax^2 + bx + c = 0$

$a = 3 \ ,\ b = -5 \ ,\ c = -1$

$b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-1) = 37$

$-b = -(-5) = +5$
$2a = 2 \times 3 = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5 - \sqrt{37}}{6} \quad or \quad \frac{5 + \sqrt{37}}{6}$$

$$= \frac{-1 \cdot 0827....}{6} \quad or \quad \frac{11 \cdot 0827....}{6}$$

$$x = -0 \cdot 1804.... \quad or \quad 1 \cdot 8471....$$

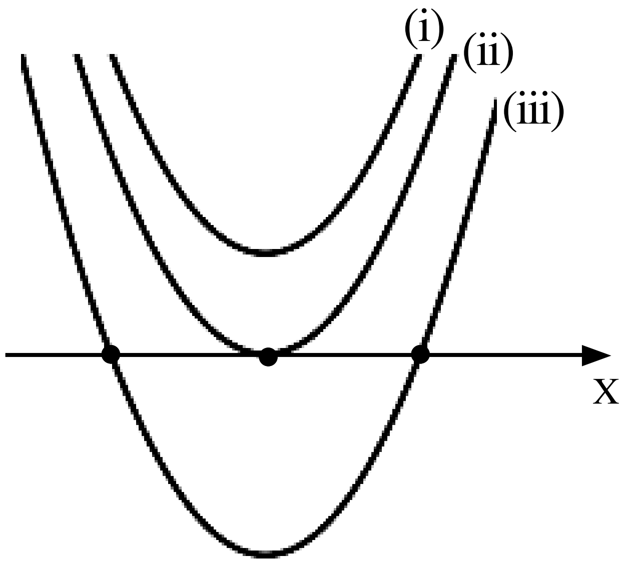
$\underline{\underline{\text{roots are } -0 \cdot 18 \text{ and } 1 \cdot 85}}$

DISCRIMINANT
$b^2 - 4ac$ gives the nature of the roots

(i) $b^2 - 4ac < 0$ NO REAL ROOTS
ie. negative

(ii) $b^2 - 4ac = 0$ ONE REAL ROOT (repeated)
(two EQUAL roots)

(iii) $b^2 - 4ac > 0$ TWO REAL ROOTS
ie. positive (and DISTINCT)



Find k so that $2x^2 - 4x - k = 0$ has **real** roots.

$$2x^2 - 4x - k = 0$$
$$ax^2 + bx + c = 0$$
$$a = 2 \ , \ b = -4 \ , \ c = -k$$
$$b^2 - 4ac = (-4)^2 - 4 \times 2 \times (-k)$$
$$= 16 + 8k$$

for real roots , $b^2 - 4ac \geq 0$

$$16 + 8k \geq 0$$
$$8k \geq -16$$
$$k \geq -2$$

FACTORISATION

RATIONAL ROOTS (non-surds): $b^2 - 4ac =$ a square number ie. $0, 1, 4, 9, 16 .....$ the quadratic can be factorised to solve the equation.

IRRATIONAL ROOTS (surds): $b^2 - 4ac \neq$ a square number solve the equation by formula.

Solve:

(1) $4n - 2n^2 = 0$

$$2n(2 - n) = 0$$
$$2n = 0 \quad \text{or} \quad 2 - n = 0$$
$$\underline{\underline{n = 0 \quad \text{or} \quad n = 2}}$$

(2) $2t^2 + t - 6 = 0$

$$(2t - 3)(t + 2) = 0$$
$$2t - 3 = 0 \quad \text{or} \quad t + 2 = 0$$
$$2t = 3$$
$$\underline{\underline{t = \frac{3}{2} \quad \text{or} \quad t = -2}}$$

The equation may need to be rearranged:

(3) $(w + 1)^2 = 2(w + 5)$

$$w^2 + 2w + 1 = 2w + 10$$
$$w^2 - 9 = 0$$
$$(w + 3)(w - 3) = 0$$
$$w + 3 = 0 \quad \text{or} \quad w - 3 = 0$$
$$\underline{\underline{w = -3 \quad \text{or} \quad w = 3}}$$

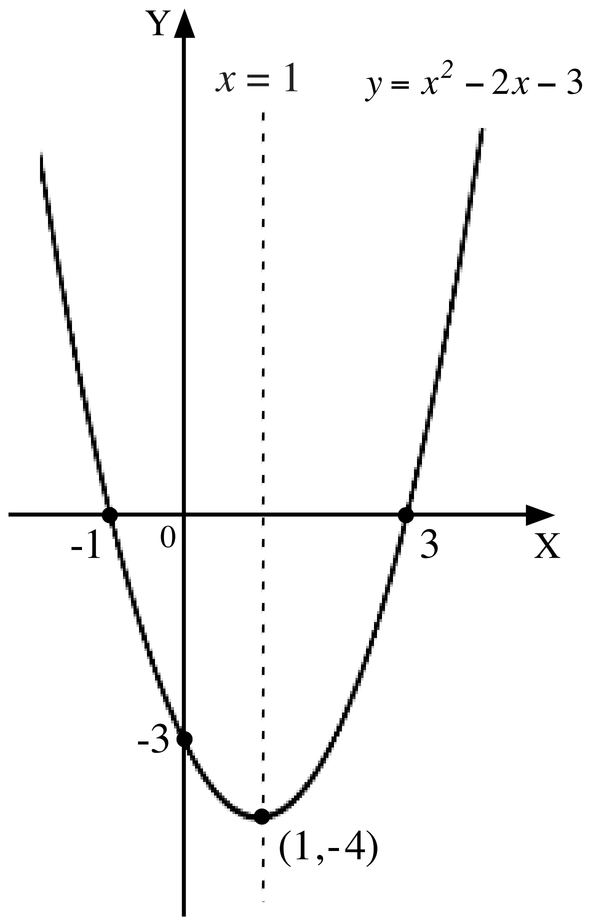
(4) $x + 2 = \frac{15}{x} \ , \ x \neq 0$

$$x(x + 2) = 15$$
$$x^2 + 2x = 15$$
$$x^2 + 2x - 15 = 0$$
$$(x + 5)(x - 3) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$
$$\underline{\underline{x = -5 \quad \text{or} \quad x = 3}}$$

GRAPHS

A sketch of the graph of a quadratic function should show where the parabola meets the axes and the maximum or minimum turning point.

Sketch the graph $y = x^2 - 2x - 3$.



(i) meets the Y-axis where x = 0

$y = 0^2 - 2 \times 0 - 3 = -3$ ***point (0,-3)***

(ii) meets the X-axis where y = 0

$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x = -1 \quad \text{or} \quad x = 3$$

points (-1,0) and (3,0)

(iii) axis of symmetry

vertical line half-way between the zeros

$x = \dfrac{-1+3}{2} = \dfrac{2}{2} = 1$, ***equation*** $x = 1$.

(iv) turning point

lies on the axis of symmetry $x = 1$

$y = 1^2 - 2 \times 1 - 3 = -4$ ***point (1,-4)***

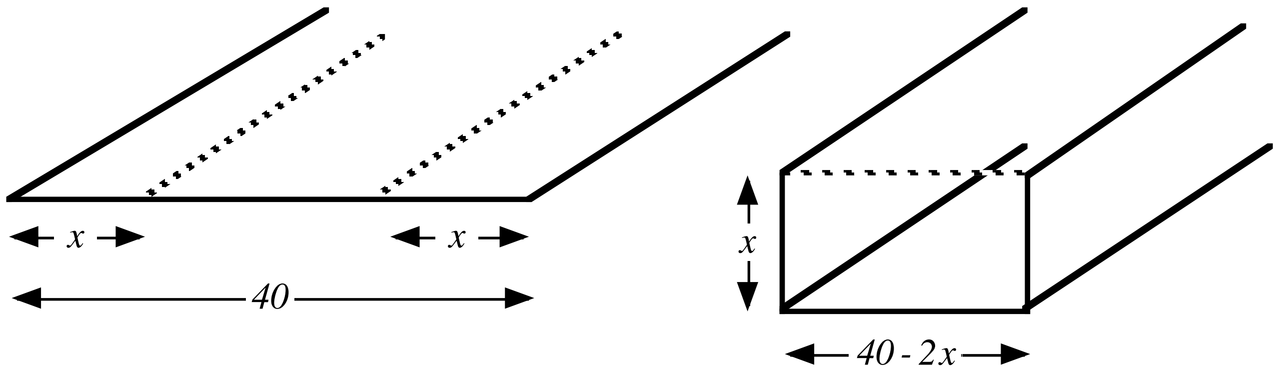
Note: (1) from the graph, (1,-4) is a **minimum turning point**.

(2) the **minimum value** of the function is -4.

APPLICATIONS

Problems involving maxima or minima which can be modelled by a quadratic equation.

A sheet of metal 40 cm. wide is folded x cm from each end to form a gutter.
To maximise water flow the rectangular cross-section should be as large as possible.

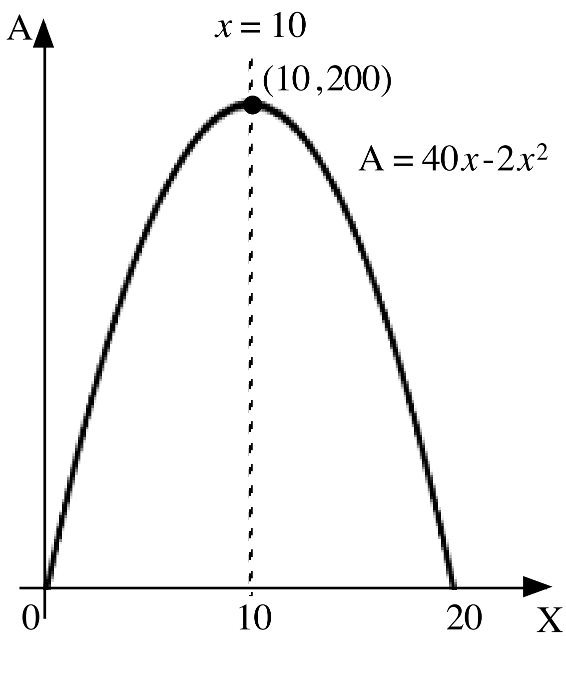


Find the maximum cross-sectional area possible.

$$A = lb$$
$$= x(40 - 2x) \qquad \textit{sketch the graph } A = 40x - 2x^2$$
$$= 40x - 2x^2$$

Zeros:

$40x - 2x^2 = 0$
$2x(20 - x) = 0$
$2x = 0$ or $20 - x = 0$
$x = 0$ or $x = 20$



Turning Point:

$(0 + 20) \div 2 = 10$
axis of symmetry $x = 10$

$y = 40x - 2x^2$
$y = 40 \times 10 - 2 \times 10^2 = 200$

maximum turning point(10,200)

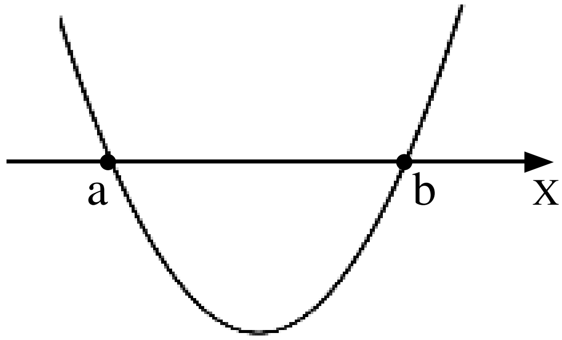
Maximum area 200 square centimetres.

EQUATION OF A GRAPH

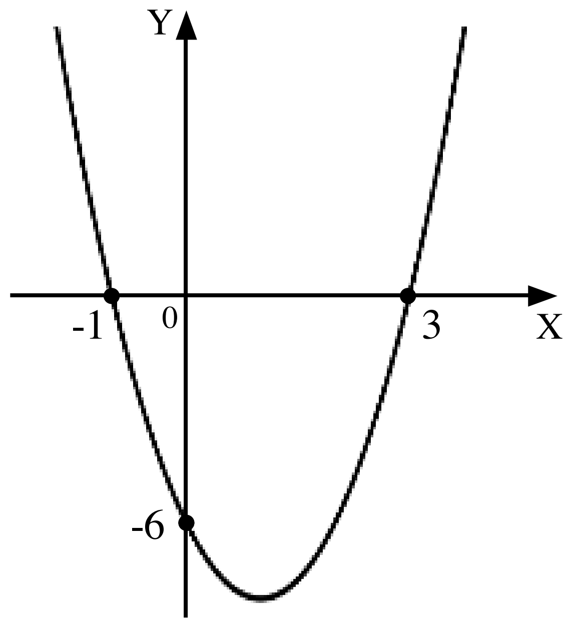
$$y = k(x-a)(x-b)$$

a and b are the zeros of the graph

k is a constant.



Write the equation of the graph.



$$y = k(x-a)(x-b)$$

zeros -1 and 3

$$y = k(x+1)(x-3)$$

for $\overset{x \quad y}{(0,-6)}$, $\quad -6 = k(0+1)(0-3)$

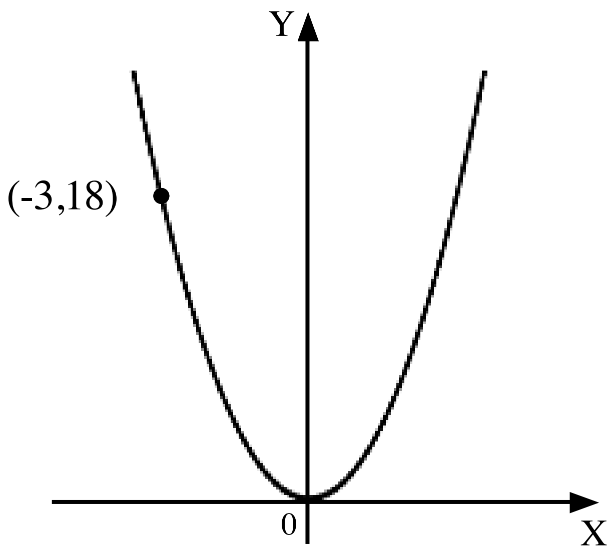
$$-6 = k \times 1 \times (-3)$$

$$-6 = -3k$$

$$k = 2$$

$$y = 2(x+1)(x-3)$$

Write the equation of the graph.



$$y = kx^2$$

for $\overset{x \quad y}{(-3,18)}$, $\quad 18 = k \times (-3)^2$

$$18 = 9k$$

$$k = 2$$

$$y = 2x^2$$

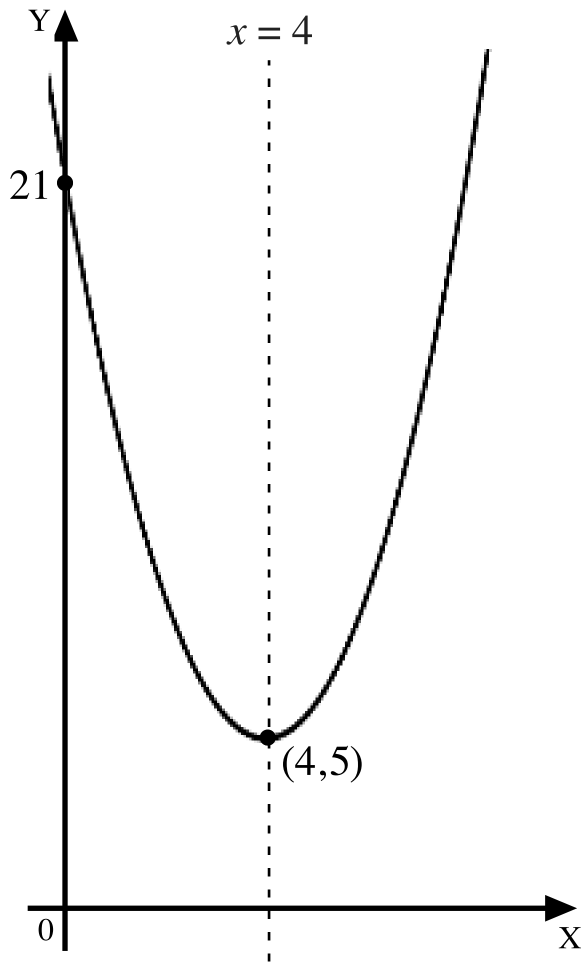
COMPLETED SQUARE:

Quadratic functions written in the form $y = \pm 1(x-a)^2 + b$, a and b are constants.

axis of symmetry $\quad x = a$

turning point $\quad (a, b)$, minimum for $+1$, maximum for -1

$y = x^2 - 8x + 21$ can be written as $y = (x-4)^2 + 5$



meets the y-axis where $x = 0$

$y = (0-4)^2 + 5 \;=\; 16 + 5 \;=\; 21$

point $(0, 21)$

turning point: $y = +1(x-4)^2 + 5$

minimum turning point $(4,5)$

axis of symmetry $\quad x = 4$

COMPLETING THE SQUARE:

$x^2 - 8x + 21$

$x^2 - 8x \qquad\qquad + 21$ *coefficient of x* , -8 , *is halved and squared* , $(-4)^2$ *to* 16

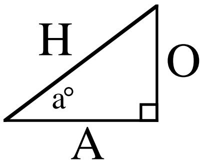
$= x^2 - 8x + 16 \quad - 16 \;+ 21$ *add and subtract* 16

$= \quad (x-4)^2 \qquad - 16 \;+ 21$ *complete square around trinomial*

$= \quad (x-4)^2 + 5$

CHAPTER 16: TRIGONOMETRY: RIGHT-ANGLED TRIANGLES

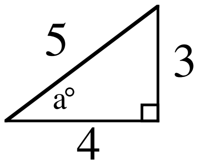
SOH-CAH-TOA The sides of a right-angled triangle are labelled:



Opposite: opposite the angle a°.
Adjacent: next to the angle a°.
Hypotenuse: opposite the right angle.

The ratios of sides $\frac{O}{H}, \frac{A}{H}$ and $\frac{O}{A}$ have values which depend on the size of angle a°.
These are called the sine, cosine and tangents of a°, written sin a° , cos a° and tan a°.

For example,



$$S = \frac{O}{H} \qquad C = \frac{A}{H} \qquad T = \frac{O}{A}$$

$$\sin a^\circ = \frac{3}{5} \qquad \cos a^\circ = \frac{4}{5} \qquad \tan a^\circ = \frac{3}{4}$$

The trig. function acts on an angle to produce the value of the ratio.
The inverse trig. function acts on the value of a ratio to produce the angle.

For example,

$$\sin 30^\circ = 0.5$$

$$\sin^{-1} 0.5 = 30^\circ$$

ACCURACY
Rounding the angle or the value in a calculation can result in significant errors.

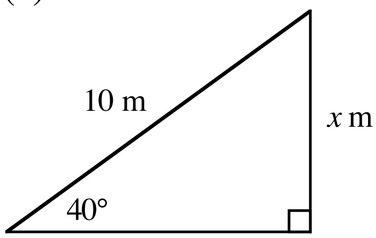
For example,

$100 \times \tan 69.5^\circ = 267.462... \approx 267$ $\qquad$ $\tan^{-1} 2.747 = 69.996... \approx 70.0$

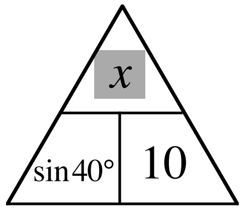
$100 \times \tan 70^\circ = 274.747... \approx 275$ $\qquad$ $\tan^{-1} 2.7 = 69.676... \approx 69.7$

FINDING AN UNKNOWN SIDE

(1) Find x.



rearrange for x

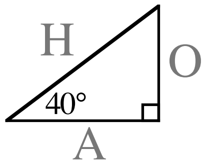


$$\sin 40^\circ = \frac{x}{10}$$

$$x = 10 \times \sin 40^\circ$$ *ensure calculator set to **DEGREES***

$$= 6 \cdot 427....$$

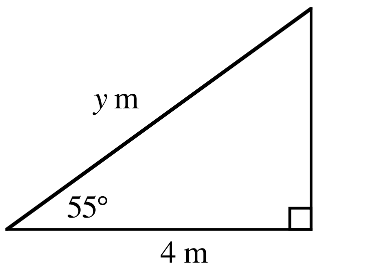
$$x = 6 \cdot 4$$



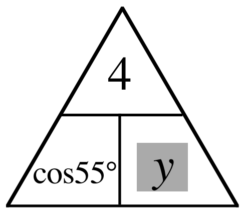
know H , find O

SOH-CAH-TOA

(2) Find y.



rearrange for y

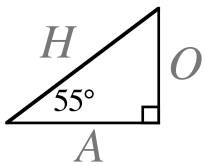


$$\cos 55^\circ = \frac{4}{y}$$

$$y = \frac{4}{\cos 55^\circ}$$ *4 ÷ cos55°, calculator set to DEGREES*

$$= 6 \cdot 973....$$

$$y = 7 \cdot 0$$

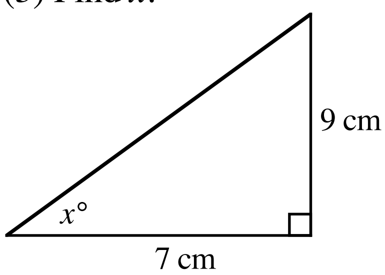


know A , find H

SOH-CAH-TOA

FINDING AN UNKNOWN ANGLE

(3) Find x.

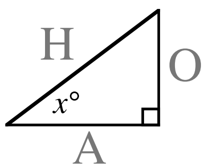


$$\tan x^\circ = \frac{9}{7}$$

$$x = \tan^{-1}\left(\frac{9}{7}\right)$$ *use brackets for (9 ÷ 7), calculator set to DEGREES*

$$= 52 \cdot 125....$$

$$x = 52 \cdot 1$$

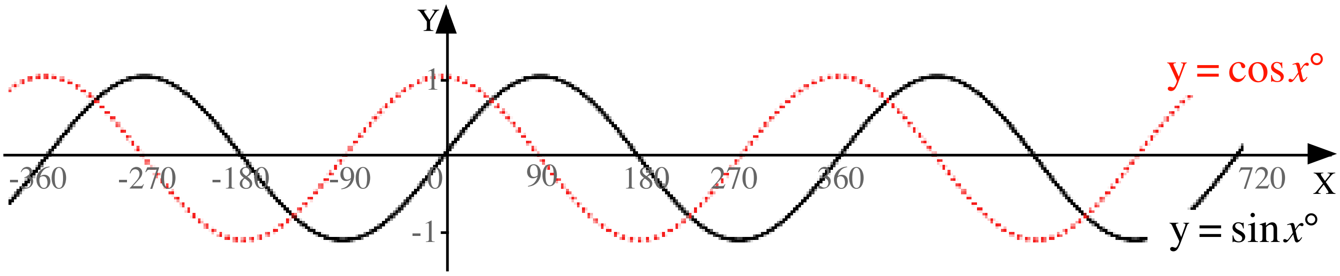


know O , know A

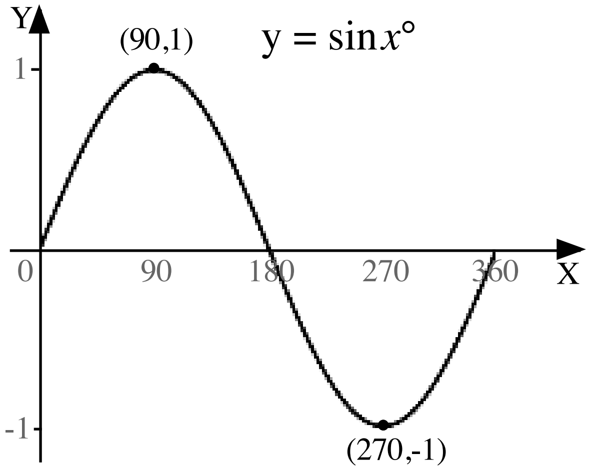
SOH-CAH-TOA

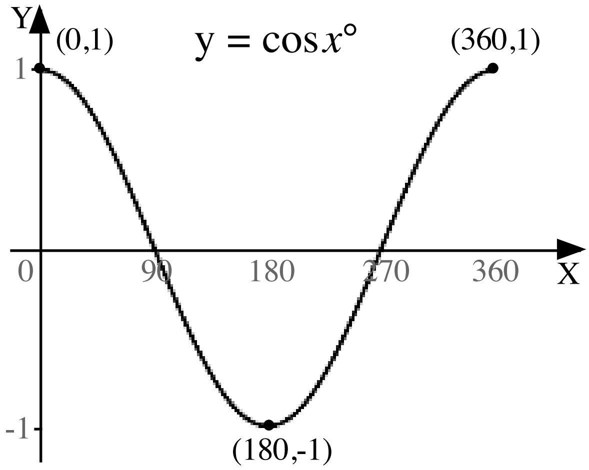
CHAPTER 17: TRIGONOMETRY: GRAPHS & EQUATIONS

The cosine graph is the sine graph shifted 90° to the left.



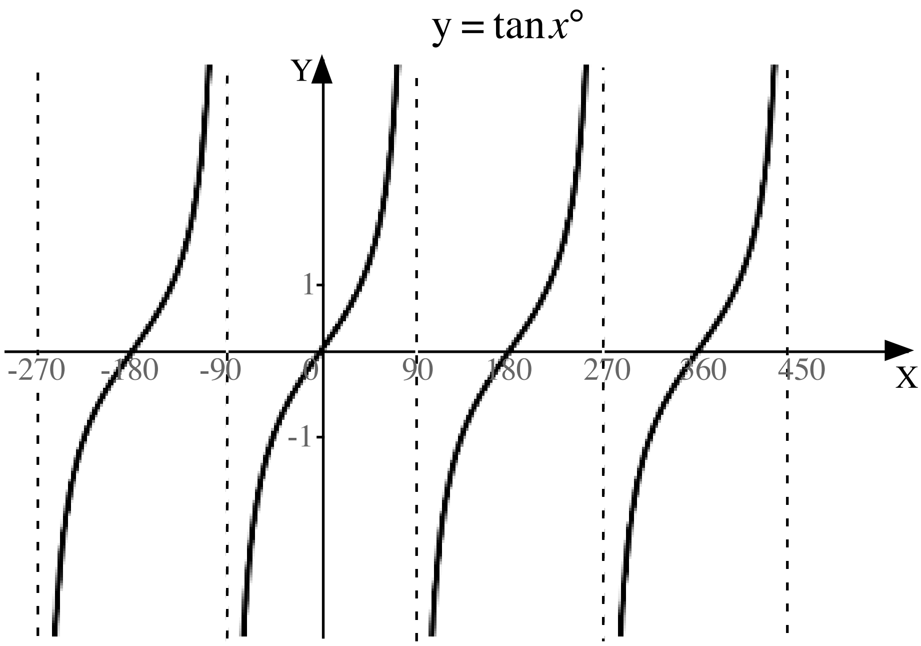
The graphs have a PERIOD of 360° (repeat every 360°).





Turning points:
maximum (90,1) , minimum (270,-1)

maximum (0,1) , minimum (180,-1)

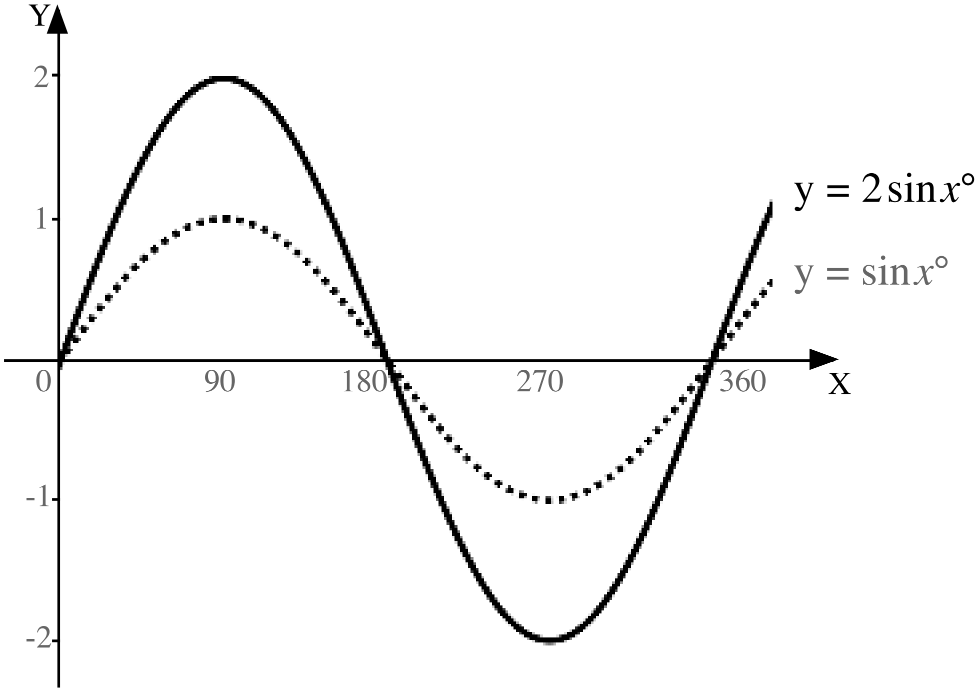


The tangent graph has a PERIOD of 180°.

TRANSFORMATIONS Same rules for $y = \sin x°$ and $y = \cos x°$.

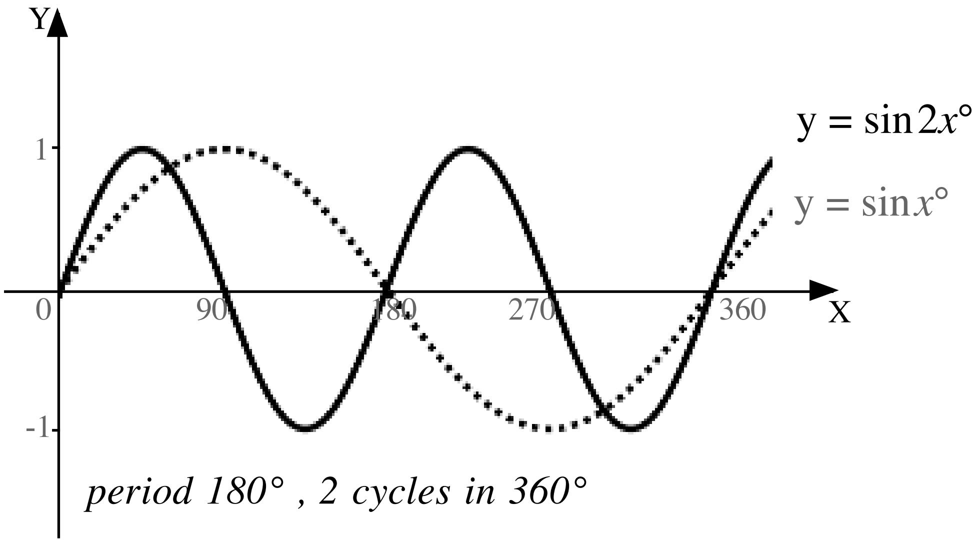
Y-STRETCH $\quad y = \mathbf{n}\sin x°$ $\quad$ y-coordinates multiplied by **n**.
amplitude **n** units
maximum value $+\mathbf{n}$, minimum value $-\mathbf{n}$

(1)



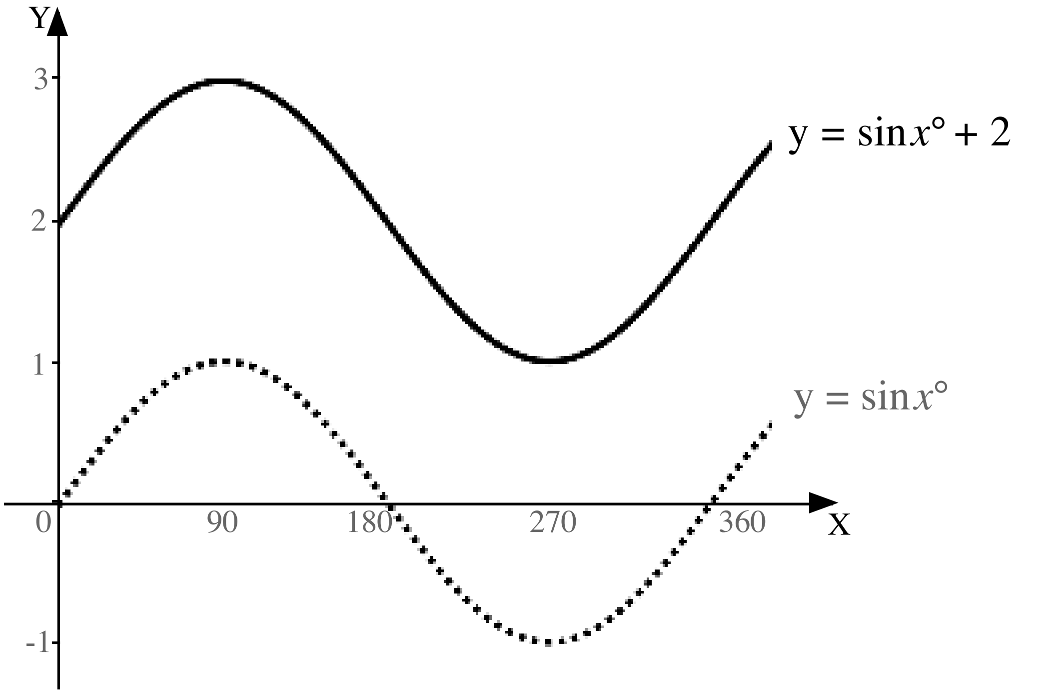
X-STRETCH $\quad y = \sin \mathbf{n}x°$ $\quad$ x-coordinates divided by **n**.
period $360° \div \mathbf{n}$. There are **n** cycles in 360°.

(2)



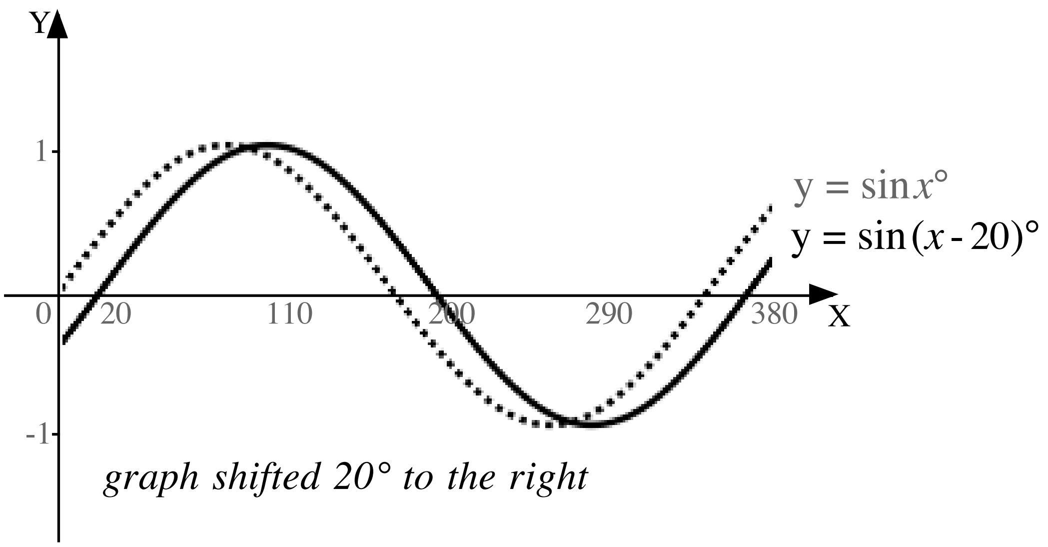
Y-SHIFT $y = \sin x° + \mathbf{n}$ add **n** units to y-coordinates
graph shifted **n** units vertically.

(3)



X-SHIFT $y = \sin(x + \mathbf{n})°$ subtract **n** units from the x-coordinates
graph shifted $-\mathbf{n}°$ horizontally.

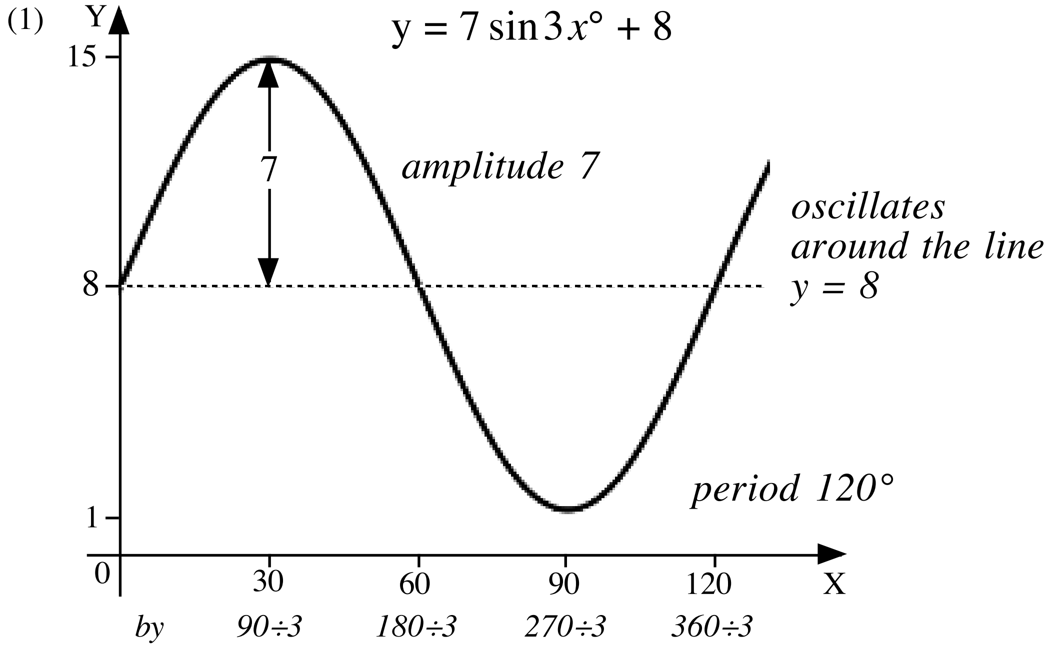
(4)



NOTE: for $y = \sin(x + 20)°$ the graph $y = \sin x°$ would be shifted 20° to the left.

COMBINING TRANSFORMATIONS

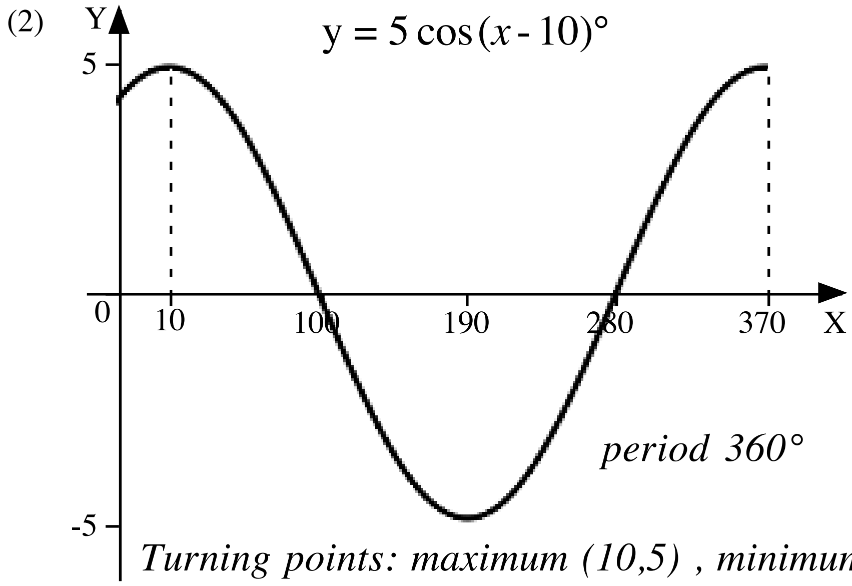
(1)



Turning points: maximum (30,15) , minimum (90,1)

$y = \sin x°$	$(90\ ,\ 1)$	$(270\ ,\ -1)$
	$\div 3 \downarrow \quad \downarrow \times 7 + 8$	$\div 3 \downarrow \quad \downarrow \times 7 + 8$
$y = 7\sin 3x° + 8$	$(30\ ,15)$	$(90\ ,\ 1)$

(2)

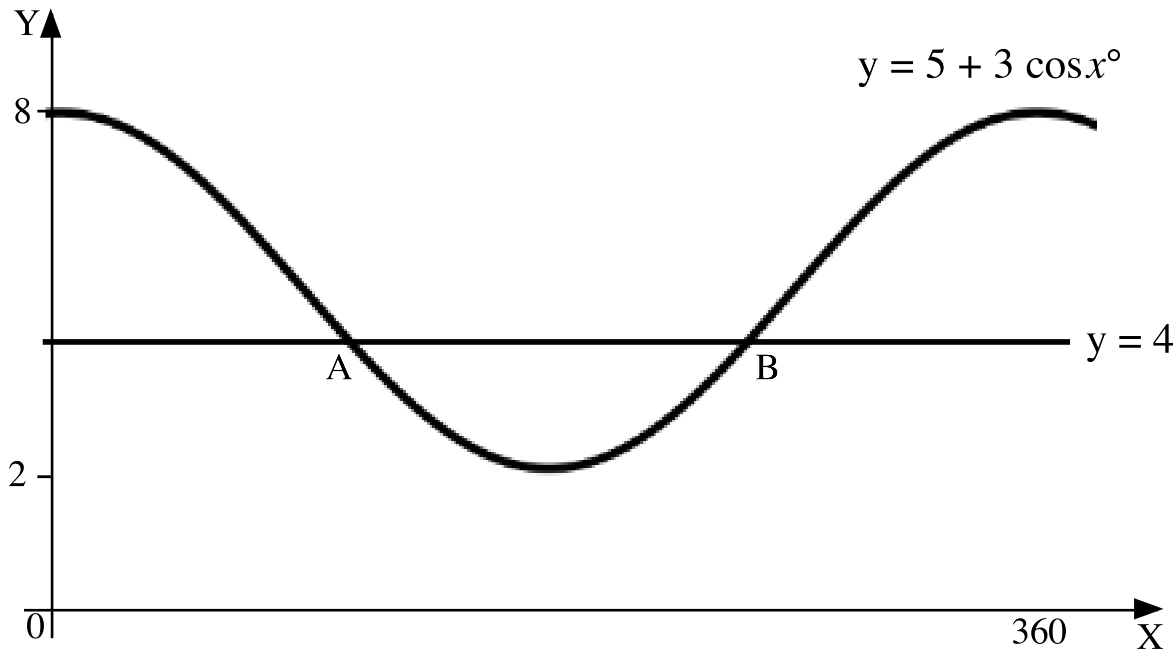


Turning points: maximum (10,5) , minimum (190,-5)

$y = \cos x°$	$(0\ ,\ 1)$	$(180\ ,\ -1)$
	$+10 \downarrow \quad \downarrow \times 5$	$+10 \downarrow \quad \downarrow \times 5$
$y = 5\cos(x\text{-}10)°$	$(10\ ,\ 5)$	$(190\ ,-5)$

EQUATIONS

The graphs with equations y = 5 + 3 cos x° and y = 4 are shown.
Find the x coordinates of the points of intersection A and B.



$$5 + 3\cos x^\circ = 4$$
$$3\cos x^\circ = -1$$
$$\cos x^\circ = -\frac{1}{3}$$

$$\underline{\underline{x = 109 \cdot 5 \ \ or \ \ 250 \cdot 5}}$$

* **A**, **S**, **T**, **C** is where functions are **positive**:

S	A
cosine negative ✓	*cosine positive* ×
180 - a = 109·5	a = cos $^{-1}$ $^1/_3$ = 70·528...
180 + a = 250·5	360 - a = 289·5
cosine negative ✓	*cosine positive* ×
T	C

* A all functions are positive
S sine function only is positive
T tangent function only is positive
C cosine function only is positive

IDENTITIES

$\sin^2 x° + \cos^2 x° = 1$

$\tan x° = \dfrac{\sin x°}{\cos x°}$

Simplify $\dfrac{1 - \cos^2 x°}{\sin x° \cos x°}$.

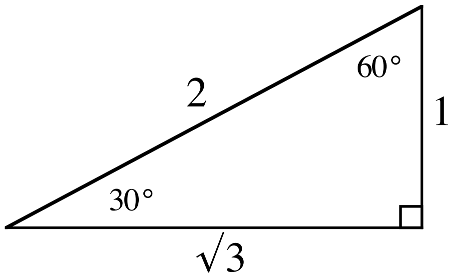
$$= \frac{\sin^2 x°}{\sin x° \cos x°} \qquad \textit{since} \quad \sin^2 x° + \cos^2 x° = 1, \quad \sin^2 x° = 1 - \cos^2 x°$$

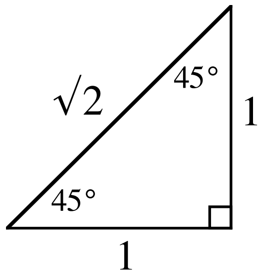
$$= \frac{\sin x° \sin x°}{\sin x° \cos x°} \qquad \text{"}\textit{cancel}\text{"} \quad \sin x°$$

$$= \frac{\sin x°}{\cos x°}$$

$$= \tan x°$$

EXACT VALUES





For example,

$\sin 60° = \dfrac{\sqrt{3}}{2}$, $\tan 30° = \dfrac{1}{\sqrt{3}}$ $\cos 45° = \dfrac{1}{\sqrt{2}}$, $\tan 45° = \dfrac{1}{1} = 1$

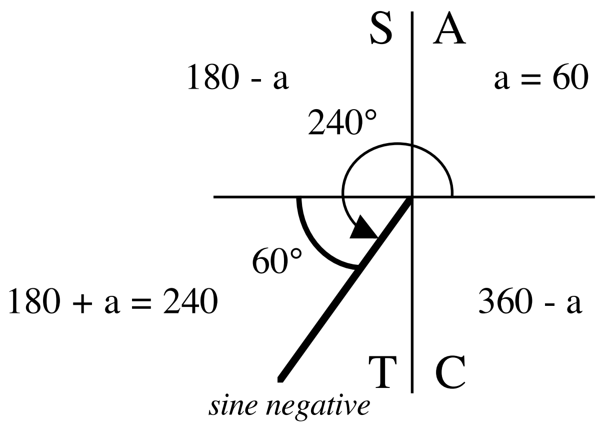
ANGLES > 90°

$\sin 240°$

$= \sin(180° + 60°)$

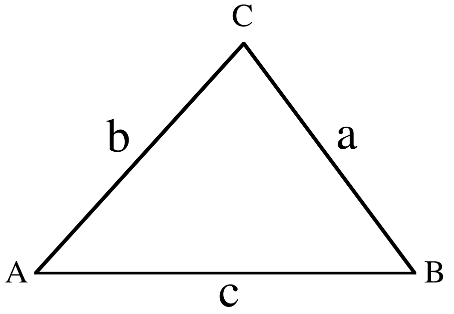
$= -\sin 60°$

$= -\dfrac{\sqrt{3}}{2}$



CHAPTER 18: TRIGONOMETRY: TRIANGLE FORMULAE

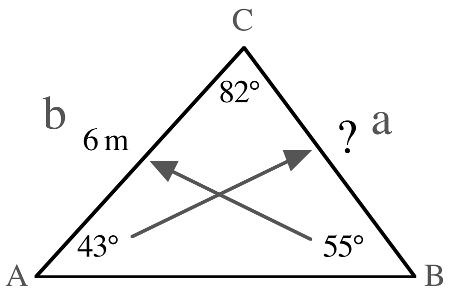
SINE RULE



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOTE: requires at least one side and its opposite angle to be known.

FINDING AN UNKNOWN SIDE



Find the length of side BC

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

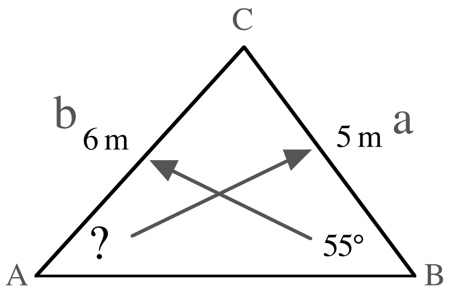
$$\frac{a}{\sin 43°} = \frac{6}{\sin 55°}$$

$$a = \frac{6}{\sin 55°} \times \sin 43°$$

$$= 4 \cdot 995.....$$

$$BC \approx 5 \cdot 0\ m$$

FINDING AN UNKNOWN ANGLE



Find the size of angle BAC.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{5} = \frac{\sin 55°}{6}$$

$$\sin A = \frac{\sin 55°}{6} \times 5$$

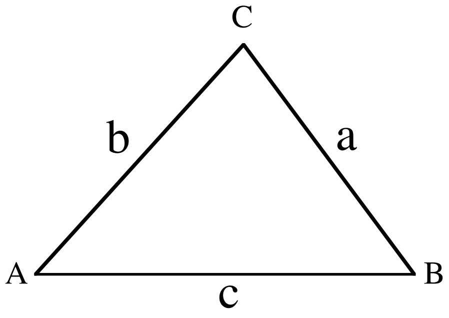
$$= 0 \cdot 682.....$$

$$A = \sin^{-1} 0 \cdot 682.....$$

$$= 43 \cdot 049.....$$

$$\angle BAC \approx 43.0°$$

COSINE RULE

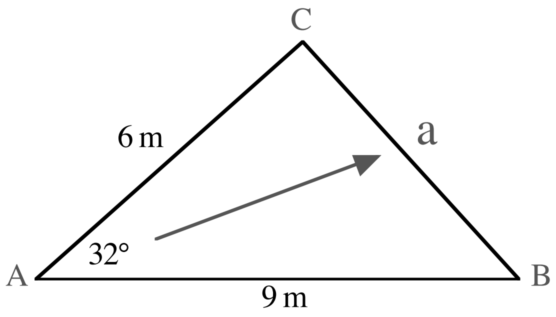


$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

FINDING AN UNKNOWN SIDE

NOTE: requires knowing 2 sides and the angle between them.

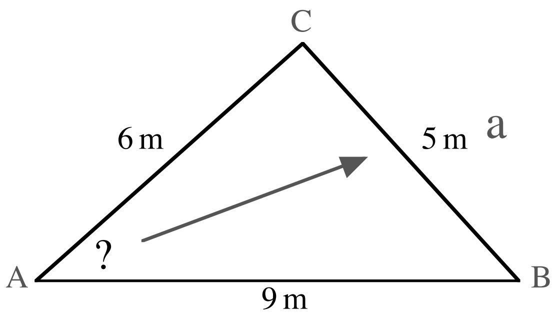


Find the length of side BC.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ &= 6^2 + 9^2 - 2\times 6\times 9\times \cos 32^\circ \\ a^2 &= 25\cdot 410..... \\ a &= \sqrt{25\cdot 410.....} \\ &= 5\cdot 040..... \\ BC &\approx 5\cdot 0\ m \end{aligned}$$

FINDING AN UNKNOWN ANGLE

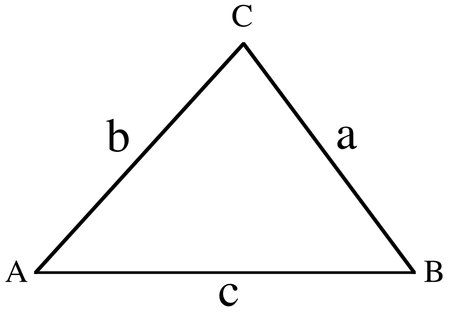
NOTE: requires knowing all 3 sides.



Find the size of angle BAC.

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{6^2 + 9^2 - 5^2}{2\times 6\times 9} \\ \cos A &= 0\cdot 85185..... \\ A &= \cos^{-1}(0\cdot 85185.....) \\ &= 31.586..... \\ \angle BAC &\approx 31\cdot 6^\circ \end{aligned}$$

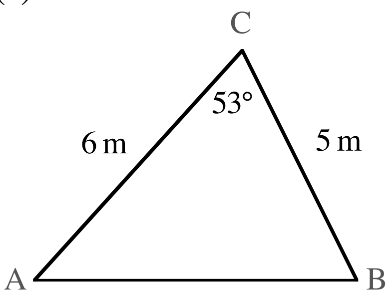
AREA FORMULA



$$Area\ \Delta ABC = \frac{1}{2}ab\sin C$$

NOTE: requires knowing 2 sides and the angle between them.

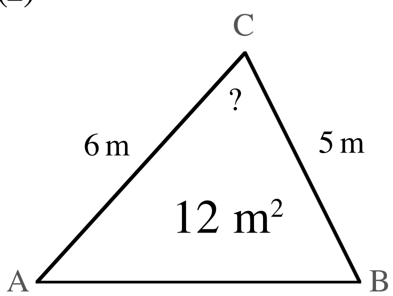
(1)



$$\begin{aligned} Area\ \Delta\ ABC &= \frac{1}{2}ab\ \sin C \\ &= \frac{1}{2}\times 5\times 6\times \sin 53° \\ &= 11\cdot 979..... \\ Area &\approx 12\cdot 0\ m^2 \end{aligned}$$

Find the area of the triangle.

(2)



$$\begin{aligned} Area\ \Delta\ ABC &= \frac{1}{2}ab\ \sin C \\ 12 &= \frac{1}{2}\times 5\times 6\times \sin C \qquad \textit{double both sides} \\ 24 &= 30\times \sin C \\ \sin C &= 24 \div 30 = 0\cdot 8 \\ C &= \sin^{-1}(0.8) \\ &= 53.130....° \quad \textit{or} \quad 126.869....° \end{aligned}$$

$(\textit{from } 180° - 53.130...°$
as angle could be obtuse)

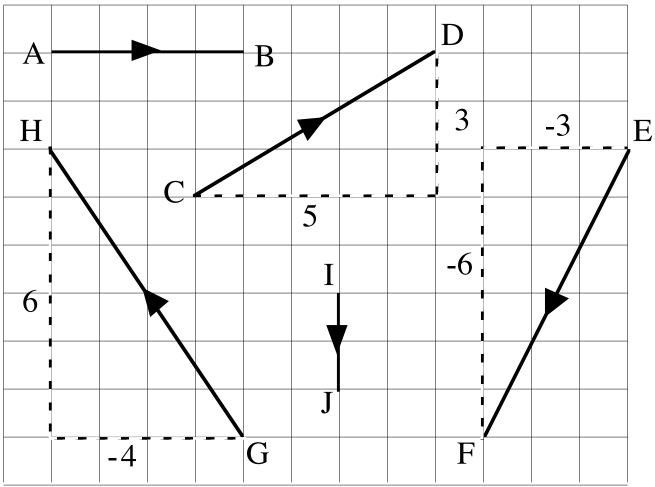
Find angle ACB.

$$\angle ACB \approx 53\cdot 1° \qquad \textit{from diagram, angle acute}$$

CHAPTER 19: VECTORS

SCALAR quantities have size(magnitude). eg. time, speed, volume
VECTOR quantities have **size** and **direction**. eg. force, velocity

A **directed line segment** represents a vector.
Vectors can be written in component form as **column vectors**



$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\overrightarrow{EF} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

$\overrightarrow{GH} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ $\overrightarrow{IJ} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

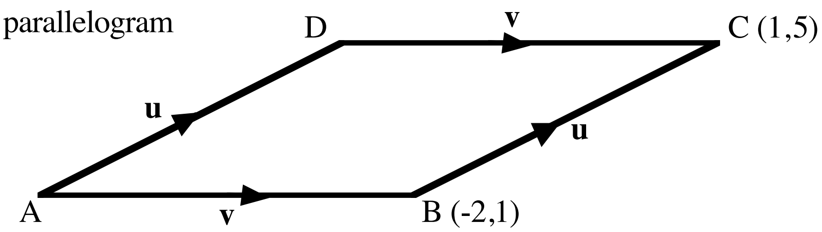
negative vector, *opposite direction*

$\overrightarrow{HG} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

SIZE follows from Pyth. Thm

$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $|\mathbf{u}| = \sqrt{a^2 + b^2}$ $|\overrightarrow{GH}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ *units*

parallelogram



$\overrightarrow{AD} = \overrightarrow{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

same size and direction
same vector **u**
same component form

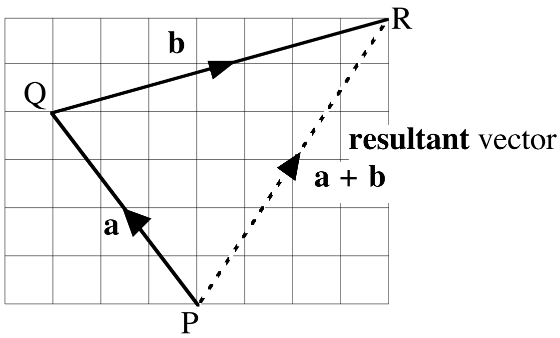
ADD/SUBTRACT

by column vectors
add or subtract components.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

by diagram
"head-to-tail" addition

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$



MULTIPLY by a number

$\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad$ $k\mathbf{u} = \begin{pmatrix} ka \\ kb \end{pmatrix}$

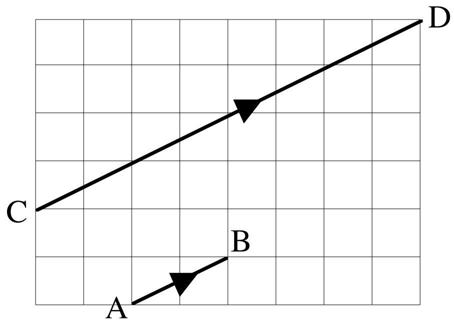
if $\quad \mathbf{v} = k\mathbf{u}$

then $\quad$ **u** *and* **v** *are parallel*

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad$ $\overrightarrow{CD} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$\overrightarrow{CD} = 4\,\overrightarrow{AB}$

CD is parallel to AB

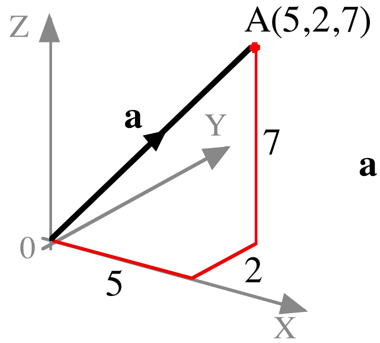


3D

Vectors in 3D operate in the same way as vectors in 2D.

Points are plotted on 3 mutually perpendicular axes.

P(x,y,z) has **position vector** $\mathbf{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



$\mathbf{a} = \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix}$

If $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ find the value of $|\mathbf{v} - 2\mathbf{u}|$.

$$\mathbf{v} - 2\mathbf{u} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix}$$

$$|\mathbf{v} - 2\mathbf{u}| = \sqrt{(-3)^2 + 4^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

CHAPTER 20: STATISTICS

Studying statistical information, it is useful to consider: (1) typical result: **average**
(2) distribution of results: **spread**

AVERAGES:

$$mean = \frac{total\ of\ all\ results}{number\ of\ results}$$

$$median = middle\ result\ of\ the\ ordered\ results$$

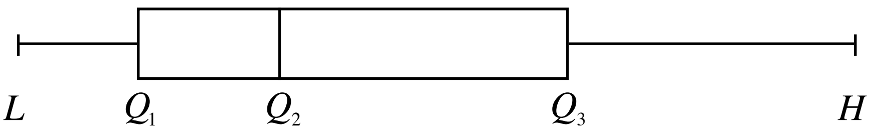
$$mode = most\ frequent\ result$$

SPREAD:

Ordered results are split into 4 equal groups so each contains 25% of the results.

The **5 figure summary** identifies: L , Q_1 , Q_2 , Q_3 , H
(lowest result , 1st , 2nd and 3rd quartiles , highest result)

A **Box Plot** is a statistical diagram that displays the 5 figure summary:



$$range,\ R = H - L$$

$$interquartile\ range,\ IQR = Q_3 - Q_1$$

$$semi-interquartile\ range,\ SIQR = \frac{Q_3 - Q_1}{2}$$

NOTE: If Q_1 , Q_2 or Q_3 fall between two results, the mean of the two results is taken.

For example,

12 ordered results: split into 4 equal groups of 3 results

$$10 \quad 11 \quad 13 \overset{Q_1}{\vdots} 17 \quad 18 \quad 20 \overset{Q_2}{\vdots} 20 \quad 23 \quad 25 \overset{Q_3}{\vdots} 26 \quad 27 \quad 29$$

$$Q_1 = \frac{13+17}{2} = 15 \quad , \quad Q_2 = \frac{20+20}{2} = 20 \quad , \quad Q_3 = \frac{25+26}{2} = 25 \cdot 5$$

The pulse rates of school students were recorded in Biology class.
Pulse rates: 66, 64, 71, 56, 60, 79, 77, 75, 69, 73, 75, 62, 66, 71, 66 beats per minute.

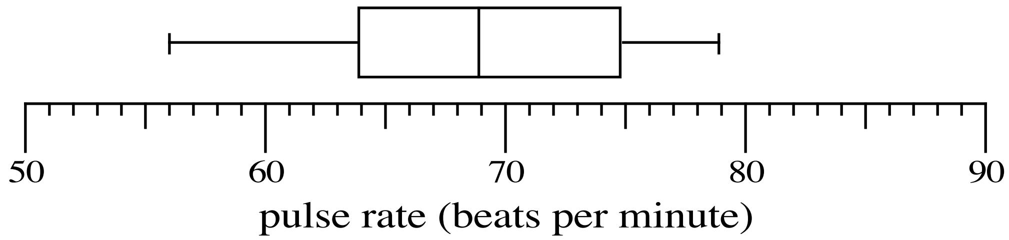
15 ordered results:

$$\begin{array}{ccccccccccccccc} & & & Q_1 & & & & Q_2 & & & & Q_3 & & & \\ 56 & 60 & 62 & \mathbf{64} & 66 & 66 & 66 & \mathbf{69} & 71 & 71 & 73 & \mathbf{75} & 75 & 77 & 79 \end{array}$$

5 Figure Summary:

$L = 56 \quad , \quad Q_1 = 64 \quad , \quad Q_2 = 69 \quad , \quad Q_3 = 75 \quad , \quad H = 79$

Box Plot:



Spread:

$$R = H - L = 79 - 56 = 23$$

$$IQR = Q_3 - Q_1 = 75 - 64 = 11$$

$$SIQR = \frac{Q_3 - Q_1}{2} = \frac{75 - 64}{2} = \frac{11}{2} = 5 \cdot 5$$

Averages: $(total = 66 + 64 + 71 + ... + 66 = 1030)$

$$MEAN = \frac{1030}{15} = 68 \cdot 666... = 68 \cdot 7$$

$$(Q_2)MEDIAN = 69$$

$$MODE = 66$$

STANDARD DEVIATION

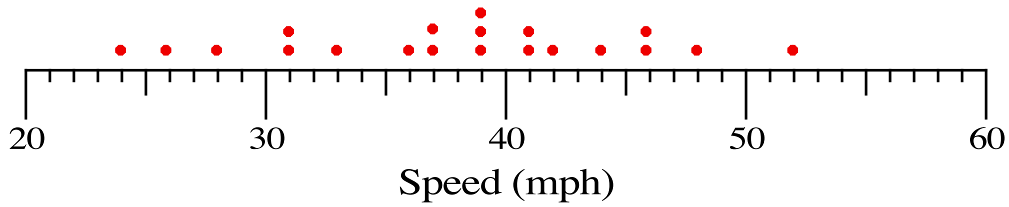
A measure of the spread of a set of data, giving a numerical value to how the data deviates from the mean. It therefore gives an indication of how good the mean is as a representitive of the data set.

Formulae:

mean $\bar{x} = \dfrac{\sum x}{n}$ standard deviation $s = \sqrt{\dfrac{\sum\left(x-\bar{x}\right)^2}{n-1}}$ or $s = \sqrt{\dfrac{\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}}{n-1}}$

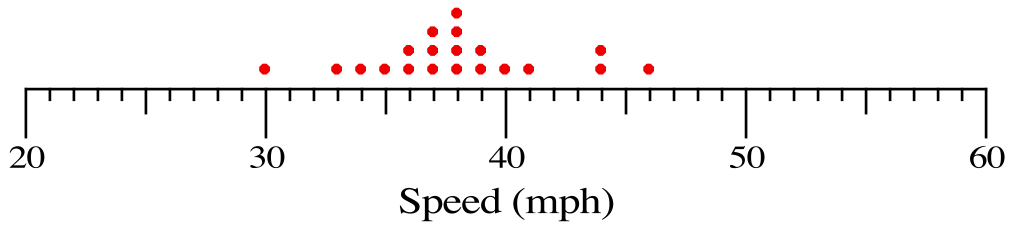
Examples,

(1) High Standard Deviation: results spread out



mean = 38 , standard deviation = 7·5

(2) Low Standard Deviation: results clustered around the mean
the results are more consistent



mean = 38 , standard deviation = 3·8

The pulse rates of 8 army recruits: $61, 64, 65, 67, 70, 72, 75, 78$ beats per minute.

$$\bar{x} = \frac{\sum x}{n} = \frac{552}{8} = 69$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
61	-8	64
64	-5	25
65	-4	16
67	-2	4
70	$+1$	1
72	$+3$	9
75	$+6$	36
78	$+9$	81
TOTALS 552		236

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{236}{7}} = 5 \cdot 806.... \approx 5 \cdot 8$$

or

$$\bar{x} = \frac{\sum x}{n} = \frac{552}{8} = 69$$

x	x^2
61	3721
64	4096
65	4225
67	4489
70	4900
72	5184
75	5625
78	6084
TOTALS 552	38324

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{38324 - \frac{552^2}{8}}{7}} = \sqrt{\frac{236}{7}} = 5 \cdot 806.... \approx 5 \cdot 8$$

PROBABILITY

The probability of an event A occuring is $$\mathrm{P}(\mathrm{A})=\frac{\text{number of outcomes involving A}}{\text{total number of outcomes possible}}$$

$0 \le \mathrm{P} \le 1$ $\qquad$ $\mathrm{P}=0$ impossible , $\mathrm{P}=1$ certain

$$\mathrm{P}(\text{not A})=1-\mathrm{P}(\mathrm{A})$$

$$\text{number of expected outcomes involving event A} = \text{number of trials} \times \mathrm{P}(\mathrm{A})$$

The experimental results will differ from the theoretical probability.

(1) In an experiment a letter is chosen at random from the word ARITHMETIC and the results recorded.

letter	frequency	relative frequency
vowel	111	111÷300 = 0·37
consonant	189	189÷300 = 0·63
	total = 300	total = 1

Estimate of probability, $\qquad P(vowel) = 0\cdot 37$

(2) A letter is chosen at random from the word ARITHMETIC.

4 vowels out of 10 letters, $\qquad P(vowel)=\frac{4}{10}=\frac{2}{5}$ $\qquad$ ie. $0\cdot 4$

for 300 trials,
$$\begin{aligned} \textit{number of vowels expected} &= \textit{number of trials} \times P(vowel) \\ &= 300 \times 0.4 \\ &= 120 \end{aligned}$$

6 consonants out of 10 letters, $\qquad P(consonant)=\frac{6}{10}=\frac{3}{5}$ $\qquad$ ie. $0\cdot 6$

$P(not\ a\ vowel) = 1-0\cdot 4 = 0.6$

vowel **or** consonant,

$P(either)=\frac{10}{10}=1$ $\qquad$ certain

vowel **and** consonant,

$P(both)=\frac{0}{10}=0$ $\qquad$ impossible